STATE OF OHIO

THE MIAMI CONSERVANCY DISTRICT

Theory of the Hydraulic Jump and Backwater Curves

BY

SHERMAN M. WOODWARD

CONSULTING ENGINEER OF THE DISTRICT, PROFESSOR MECHANICS AND HYDRAULICS, STATE UNIVERSITY OF IOWA

The Hydraulic Jump as a Means of Dissipating Energy

BY

ROSS M. RIEGEL AND JOHN C. BEEBE

TECHNICAL REPORTS
PART III

DAYTON, OHIO
1917

STATE OF OHIO

THE MIAMI CONSERVANCY DISTRICT

Theory of the Hydraulic Jump and Backwater Curves

BY

SHERMAN M. WOODWARD
Consulting Engineer of the District, Professor of Mechanics and Hydraulics, State University of Iowa

The Hydraulic Jump as a Means of Dissipating Energy

BY

ROSS M. RIEGEL AND JOHN C. BEEBE

TECHNICAL REPORTS
Part III

DAYTON, OHIO
1917
Reprinted 1928

THE MIAMI CONSERVANCY DISTRICT
DAYTON, OHIO

EDWARD A. DEEDS, Dayton Chairman HENRY M. ALLEN, Troy GORDON S. RENTSCHLER, Hamilton	Board of Directors

---

EZRA M. KUHNS, Secretary

OREN BRITT BROWN, Attorney

JOHN A. McMAHON, Counsel

ARTHUR E. MORGAN, Chief Engineer

CHAS. H. PAUL, Asst. Chief Engineer

PREFATORY NOTE

This volume is the third of a series of Technical Reports issued in connection with the planning and execution of a notable system of flood protection works in the Miami Valley.

This valley, which forms a part of the large interior plain of the central United States and comprises about 4,000 square miles of gently rolling topography in southwestern Ohio, is one of the leading industrial centers of the country. The immense damage which it sustained during the flood of March, 1913, amounting to 100 million dollars of property and over 360 lives, led to an energetic movement to prevent future disasters of this kind. This movement developed gradually into a great cooperative enterprise for the protection of the valley by one comprehensive project. The Miami Conservancy District, established in June, 1915, under the newly enacted Conservancy Law of Ohio, became the agency for securing this protection. On account of the size and character of the undertaking, the plans of the district have been developed with more than usual care.

A Report of the Chief Engineer, submitting a plan for the protection of the district from flood damage, was printed March, 1916, in 3 volumes of about 200 pages each. Volume I contains a synopsis of the data on which the plan is based, a description of its development, and a statement of the plan in detail. Volume II contains a legal description of all lands affected by the plan. Volume III contains the contract forms, specifications, and estimates of quantities and cost.

After various slight modifications this report of the Chief Engineer was adopted by the Board of Directors as the Official Plan of the District, and was republished in May, 1916.

In order to plan the project intelligently, many thorough investigations and researches had to be carried out, the results of which have proved of great value to the district and will also be of widespread value to the whole engineering profession. The object of publishing this series of Technical Reports is to make available to the residents of the state and to the technical world at large, all data of interest relating to the history, investigations, design and construction of the project.

ARTHUR E. MORGAN, Chief Engineer.

Dayton, Ohio, November 1, 1917.

LIST OF TECHNICAL REPORTS

Orders for these reports should be addressed to:
THE MIAMI CONSERVANCY DISTRICT
DAYTON, OHIO

CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

NOMENCLATURE

Page numbers refer to text where quantities are more fully defined, or where they are used.

db = change in elevation of bottom in length dl (Pages 35-51).
C = friction coefficient in Chezy formula (Page 35).
D = depth of moving stream in feet.
D_1 = initial depth of stream.
D_2 = final depth of stream.
D_c = critical depth of flow for quantity Q, $gD_c^3 = Q^2$ (Page 17).
D' = depth of uniform mixture of air and water (Page 31).
df = head required to overcome friction in length dl (Pages 35-51).
g = acceleration of gravity in feet per second.
h = elevation of water surface (Page 35).
dh = change in elevation of water surface between two consecutive cross sections (Pages 35-51).
H_1 = velocity head above jump $= \frac{V_1^2}{2g}$ (Page 73).
J $= \frac{D_2}{D_1}$ (Page 73).
k $= \frac{D_1}{D_c}$ (Page 31).
k = velocity head of moving stream (Pages 35-51).
dk = change in velocity head between two consecutive cross sections (Pages 35-51).
l = length of channel (Pages 35-51).
dl = horizontal distance between two consecutive cross sections (Pages 35-51).
Q = quantity of water flowing in channel 1 foot wide, measured in cubic feet per second (Page 15).
R = hydraulic radius of stream.
S = slope of bottom of channel.
S' = friction slope or neutral slope for a given velocity and depth (Pages 35-51).
V = velocity of moving stream in feet per second.
V_1 = initial velocity of stream.
V_2 = final velocity of stream.
V_c = critical velocity for flow Q, $V_c^3 = p$ (Page 17).
w = weight of cubic foot of water in pounds.
x $= \frac{D_1}{D_c}$ (Pages 18-26).
x $= \frac{D}{D_c}$ (Page 31).
y $= \frac{D_2}{D_c}$ (Pages 18-26).
y $= \frac{D'}{D_c}$ (Page 31).
y = depth of moving stream (Pages 35-51).
dy = change in depth between two consecutive cross sections (Pages 35-51).
y_c = critical depth of stream (Pages 35-51).
y_n = neutral depth of stream (Pages 35-51).
z $= \frac{y}{y_n}$ (Page 51).

THEORY OF THE HYDRAULIC JUMP AND BACKWATER CURVES

SECTION I.—INTRODUCTORY

The Hydraulic Jump and the Backwater Curve, two phenomena which heretofore have been but imperfectly understood, became subjects of unusual importance in connection with the design of the flood control works of the Miami Conservancy District. It was found necessary to make intensive studies covering the theoretical aspects of these phenomena, and also of their practical application to such conditions as will obtain in the operation of the dams of the District. The results of these studies are set forth in the following pages, the theoretical considerations being taken up first and the experimental data next.

The jump is to be utilized in destroying the kinetic energy of the water issuing at high velocity from the outlet conduits, and hence it was necessary to establish definitely the conditions under which it occurs and the means for its control. Not only, therefore, was the subject given a careful theoretical study, but experiments on models were also carried out for the purpose of verifying the important conclusions.

In studying this subject, the hydraulic theory has seemed more than usually elusive. Perhaps the most frequent error in hydraulic discussions, in general, is the very common one of applying a law, rule, or formula to a situation where it does not hold. Every rule in hydraulics is limited in its application to a definite set of conditions outside of which it may have no bearing whatsoever. This tendency to apply a perfectly good rule or formula to the wrong situation is naturally not only difficult to avoid in the first instance, but also not easy to refute by argument after the mistake has been made.

In this study but limited help was received from the published literature. Only scanty attention is given to the subject in the common textbooks and authorities; while the references in periodical and specialized literature are widely scattered and sometimes contain errors of the sort mentioned above. Although the phenomena, as shown later, rest almost exclusively upon the most

universally recognized laws of mechanics and hydraulics, the difficulty of obtaining a clear and convincing statement covering them, the prevalence of errors in references to and discussions of them, together with their importance in this particular situation, make it appear desirable to attempt a full and complete exposition of the whole field.

The attempt will be made to take up the simplest cases first, and to proceed, one step at a time, to the most complex cases, giving a complete explanation of each step together with physical descriptions of all phenomena, independent, so far as possible, of mathematical symbolism. At the same time the mathematical treatment will be fully given or indicated. It is believed, however, that it is entirely possible to gain a complete understanding of all the physical phenomena concerned, without following through any complicated mathematical work. Diagrams are used freely to indicate the changes in the various quantities.

SECTION II.—SUMMARY

In a prismatic, frictionless open channel with level bottom it is impossible, according to Bernouilli's theorem, for a moving stream of water to change its depth of flow. An apparent exception to this rule exists when the flow is at the critical stage, that is when $V^2 = gD$, or when $V^3 = gQ$. At this stage the velocity head equals one-half the depth, and according to the theory, change would be possible. However, as soon as the depth is varied the slightest amount, further change becomes impossible. Hence, practically, the critical stage is no exception to the general rule.

It is possible to produce a change in the depth of flow, or in the elevation of the surface, by varying the cross section, either by giving the bottom an inclination or slope, or by changing the width. Friction produces a change somewhat analogous to a negative bottom slope.

When the velocity is accelerated by the conversion of pressure into velocity, it is impossible for internal impact to occur during the transformation, or for the change to take place otherwise than in accordance with Bernouilli's theorem. When the change takes place in the reverse direction, however, that is, when the velocity is being reduced, internal impact is likely to occur and can be prevented only by especial care. Any such internal im-

pact destroys kinetic energy and modifies the applicability of Bernouilli's theorem.

For a channel open to the air, the theory of one such method of reducing velocity by internal impact was worked out by the Frenchman, Belanger, in 1838. The striking physical phenomenon which exists in such a case seems appropriately denominated the *hydraulic jump*. Belanger's theoretical analysis of this phenomenon seems to have been abundantly verified by numerous experiments.

The jump can occur only when the initial velocity is greater than, and the depth of flow correspondingly smaller than, the values for critical flow. The jump takes place across the critical point so that the depth subsequent to the jump is greater than the critical value. The change in depth is definitely determined by the initial values of depth and velocity, and the depth subsequent to the jump is given by the equation

$$D_2 = -\frac{D_1}{2} + \sqrt{\frac{2V_1^2 D_1}{g} + \frac{D_1^2}{4}}$$

A hypothesis is given regarding the condition existing within the jump, which suffices to account for the possibility of its existence.

A backwater curve is defined as the curve taken by the surface of a stream of water in which all changes of velocity take place in accordance with Bernouilli's theorem. This includes the effect of ordinary friction where the retarding force produces a smoothly distributed and regular effect, but excludes cases of sudden impact, such as exist in the hydraulic jump.

Neutral depth of flow in a channel is defined as the depth at which the bottom slope is equal to the slope required to overcome friction. It is the depth, therefore, at which the depth and velocity may remain unchanged; or, in other words, the depth for stable uniform flow.

All the possible backwater curves for a wide rectangular channel are considered in detail. Twelve different cases are distinguished according to the relative magnitude of the critical depth, the neutral depth, and the actual depth. For each case the internal conditions are completely analyzed and finally a

typical backwater curve for each case is shown plotted accurately to scale by the use of Bresse's tables and some supplementary calculations.

The application of backwater curves and the hydraulic jump to actual conditions is shown by two simple instances illustrating transition backwater curves and the necessary existence of the hydraulic jump to connect two backwater curves of different types.

SECTION III.—BERNOUILLI'S THEOREM APPLED TO A FRICTIONLESS OPEN CHANNEL

Consider first, water flowing along a horizontal rectangular channel with a high velocity, and that by a smooth gradual enlargement in the cross section the velocity is to be reduced. Assume that the effect of friction may be ignored. Bernouilli's theorem, then, states that if water moves in such a stream from one point to another at the same level, without loss of energy by friction or impact, the sum of the static head and the velocity head at the first point is equal to the sum of the same quantities at the second point.

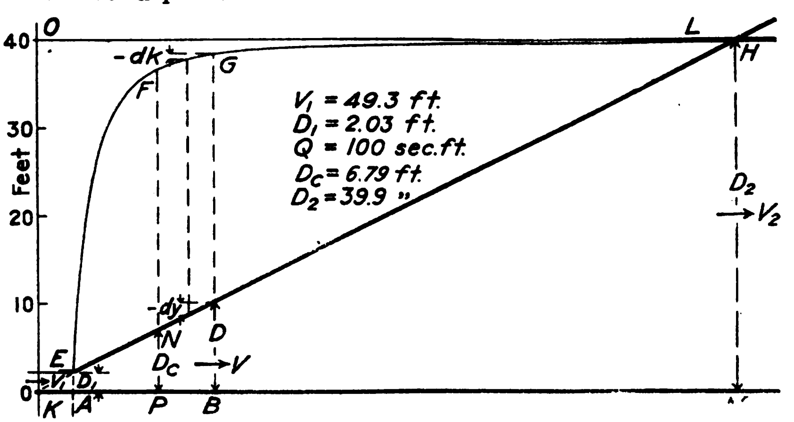


FIG. 1.—DIAGRAM ILLUSTRATING FLOW IN AN EXPANDING TUBE.

Constant width, level bottom, no friction or impact. The ordinates of the curve *EFGH* represent the static pressure on the bottom *AM;* the height of the curve *EFGH* above the straight line *EH* represents the static pressure on the top *EH;* and the distances of the curve *EFGH* below the line *OL* represent the head corresponding to the velocity within the tube. The critical conditions of flow exist at the section *NP*.

Let figure 1 represent such a condition. Suppose the original velocity is V_1 and depth D_1. If the channel is open at the top,

the static head at the point A is D_1 and the velocity head is $V_1^2/2g$.

Suppose the channel to be of uniform width, and the bottom level. In order to secure the gradual smooth enlargement of cross section, suppose a rigid top to be added to the channel as shown from E to H, rising along a straight line. We know by experience that if the angle is not made too great, the water will cling to the inclined plane and as the cross section increases the velocity will be smoothly and gradually reduced, as exemplified in the expanding tube of the Venturi meter. At some point as B, where the depth has increased to D and the velocity has decreased to V, the velocity head would have diminished to $V^2/2g$.

Since the quantity of water flowing past each point is the same, let $Q =$ volume of water per foot in width of the conduit, then

$$Q = V_1 D_1 = VD \tag{1}$$

or

$$V = \frac{D_1 V_1}{D} \tag{2}$$

By Bernouilli's theorem the static pressure at B then equals

$$D_1 + \frac{V_1^2}{2g} - \frac{V^2}{2g} = D_1 + \frac{V_1^2}{2g}\left(1 - \frac{D_1^2}{D^2}\right) \tag{3}$$

In general, this pressure at B would not exactly equal D, but might be something greater, as BG. By means of the above expression, the static pressure corresponding to each depth can be easily calculated for given conditions. This has been done for figure 1, which has been drawn for the case in which $D_1 = 2.03$ feet, $V_1 = 49.3$ feet per second, whence $Q = D_1 V_1 = 100$ second feet, and $V_1^2/2g = 38$ feet, using $g = 32$.

The resulting static pressures, when plotted above the base AM, give the curve $EFGH$. This curve, which crosses the surface of the water at E and H, indicates relations of great fundamental importance which should be carefully noted. At E the static pressure on the bottom is exactly that due to the depth of water AE. As soon as the velocity is reduced and part of the velocity head is thus converted into pressure, the pressure on the

bottom is greater than that due to the depth of the water alone. Thus at P the pressure is that caused by a head FP. This means that at the point N there is a pressure against the upper bounding surface of the water equal to that caused by a head FN. If a small piezometer tube should be inserted through the upper surface at N, the water would rise in it to the level of F. This pressure tends to burst the cover off the conduit. The curve $EFGH$ is really the hydraulic grade line through the expanding section.

This interior bursting pressure against the upper surface begins with zero at E and gradually increases until it reaches a maximum value such as FN at N. From this point onward it gradually decreases until it again reaches zero at H, where the pressure on the bottom M is again exactly that caused by the depth of the water.

As the velocity decreases between E and N, the corresponding diminution of velocity head is more than enough to raise the water along the rigid, slanting, upper boundary surface, and hence there is an accumulation of excess static pressure. As the velocity decreases from N to H, the gradual conversion of velocity head into pressure is not sufficient to raise the water, and hence, the previous accumulation of pressure is gradually drawn upon in raising the surface to higher and higher elevations.

If the whole apparatus is open to the air, the water at H would no longer follow the upper slanting surface, but would break loose and flow away with level surface from this point. If the air were excluded by suitable means beyond the point H, it might be possible to have the upper water surface continue to cling to the slanting face, in which case the water beyond H would be under a vacuum, increasing in intensity the farther the expansion of cross section is continued.

The gradual variation of pressure along EH is obviously reasonable from the following considerations. For a given small change in cross section, the change in velocity near E will be much greater than near H. Again, since the velocity head is proportional to the square of the velocity, for a given change in velocity, the change in velocity head will be greater as the velocity itself is greater. To illustrate, the difference between the squares of 101 and 100 is 201, while the difference between the squares of 11 and 10 is only 21. That is, a change of one foot in a velocity of 100 would have about 20 times as much effect on the velocity head as a change of one foot in a velocity of 10.

For both of these reasons, then, the pressure changes most rapidly at E.

The cross section at which the upward pressure on the surface EH is a maximum, shown on figure 1 at NP, is of particular interest. At this place a tangent to the curve at F is parallel to the water surface at N, and the change of velocity head is just sufficient to produce the change in elevation of water surface, so that there is neither accumulation nor reduction of pressure. On account of this peculiar balance between velocity head and depth, this particular section may be said to mark a critical point or a condition of critical flow. We shall find hereafter that this critical point appears in numerous important relations. Let the subscript c denote values at the critical point. From the value in equation 3, it is easily proved by the differential calculus that

$$V_c = \sqrt{gD_c} = \sqrt[3]{gQ} \tag{4}$$

and

$$D_c = \frac{V_c^2}{g} = \sqrt[3]{\frac{Q^2}{g}} \tag{5}$$

When $Q = 100$, $g = 32$, $D_c = 6.79$ feet, $V_c = 14.7$ feet per second.

From equation 4 it may be easily shown that at the critical point the velocity head is one-half the depth. This leads to a convenient definition or physical picture of the condition of critical flow. In any rectangular channel critical flow exists when the velocity head is one-half the depth of the moving stream. Thus in figure 1 at P the velocity head, represented by the vertical distance from F to the horizontal axis OL, is equal to one-half the depth, NP.

But little further discussion of figure 1 is necessary. The curve $EFGH$ has as asymptotes the straight lines OL and OK. The upper surface of the water EH need not follow a straight line. Any smooth curve would do. The curve $EFGH$ would change correspondingly in shape, but would have the same properties as enumerated above, and would, for each depth of flow, be at the same distance above the water surface, as in the case of the straight top.

The depths at E and H always have a definite relation to each

other because at these two points the sum of the depth and the velocity head is the same.

Let D_1 = depth at E

D_2 = depth at H

Then

$$\frac{V_1^2}{2g} + D_1 = \frac{V_2^2}{2g} + D_2 \qquad (6)$$

Substituting for velocities their values in terms of Q, and transposing

$$\frac{Q^2}{2gD_1^2} - \frac{Q^2}{2gD_2^2} = D_2 - D_1$$

$$\frac{Q^2}{2g}\,\frac{D_2^2 - D_1^2}{D_2^2D_1^2} = D_2 - D_1$$

Dividing through by $D_2 - D_1$, and substituting D_c^3 for Q^2/g

$$D_c^3 = \frac{D_1^2D_2^2}{\dfrac{D_1 + D_2}{2}} \qquad (7)$$

If the values of any two of the depths are given, the value of the third can readily be obtained. The equation is symmetrical in D_1 and D_2. When D_1 equals D_c, D_2 will also equal D_c. When D_1 is less than D_c, D_2 must be greater than D_c, and vice versa. On figure 2 is plotted the graph of equation 7 for $Q = 100$ and $D_c = 6.79$.

In equation 7, if x be substituted for D_1/D_c, and y for D_2/D_c, the equation becomes

$$\frac{x^2y^2}{x + y} = \frac{1}{2} \qquad (8)$$

The graph of this equation is shown on figure 3.

The method described above is not the only device by which theoretically the depth of water flowing in the rectangular channel may be varied. Suppose that instead of the rigid top applied to the conduit as shown in figure 1, the bottom be raised on a gradual ascent as shown in figure 4. The water will then flow up the incline as indicated in the figure. As the water rises

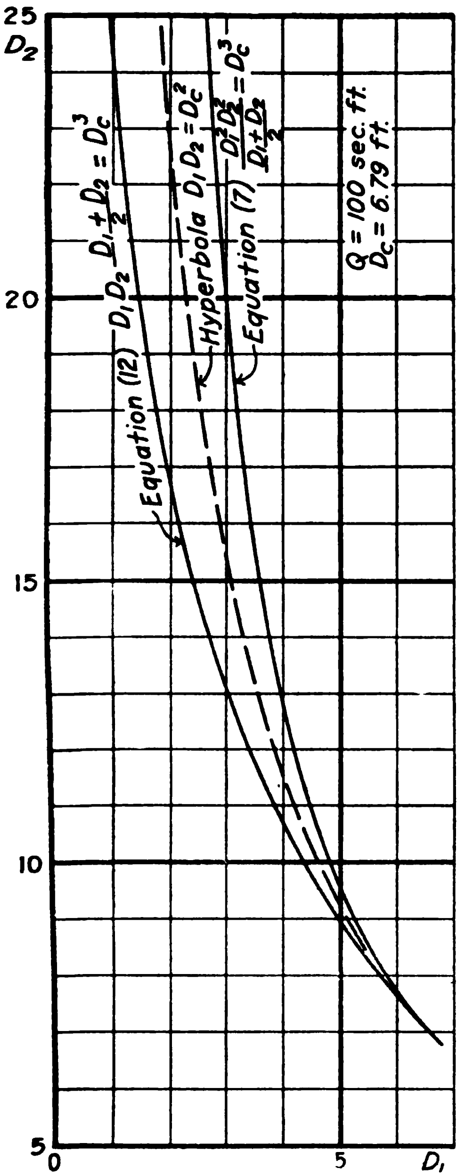


FIG. 2.—PLAT OF EQUATIONS 7 AND 12. Discharge 100 second feet, critical depth 6.79 feet.

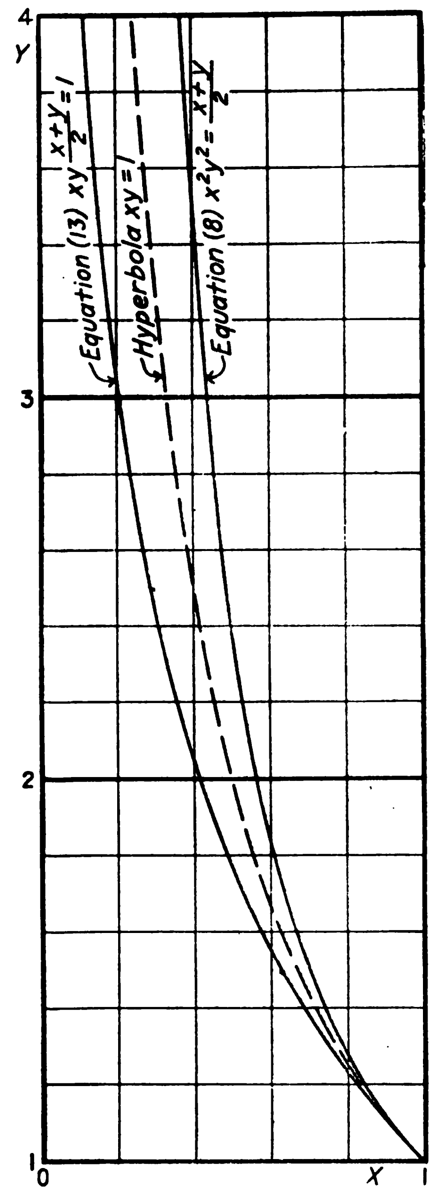


FIG. 3.—PLAT OF EQUATIONS 8 AND 13.

from E, some of the initial kinetic energy will be consumed in lifting the water. At first when the water has a high velocity, sufficient energy can be supplied with but a slight decrease in velocity. Hence, on the lower part of the slope the depth will increase very slowly. But as greater heights are reached, it

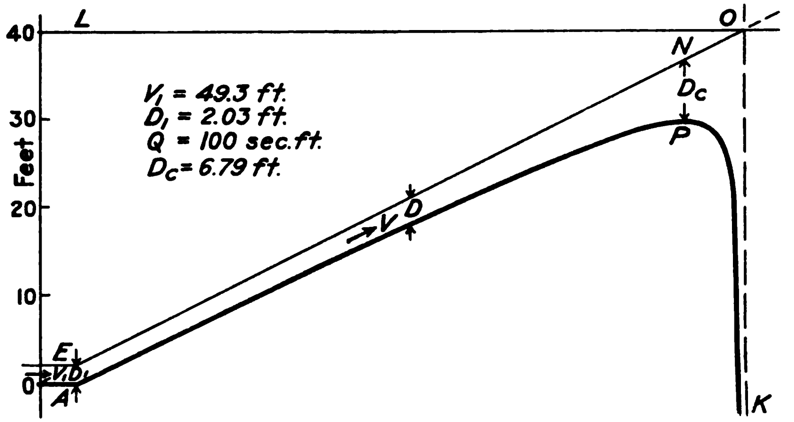


FIG. 4.—DIAGRAM ILLUSTRATING FLOW IN AN OPEN CONDUIT OF CONSTANT WIDTH.

No friction or impact. The profile of the bottom is that necessary to raise the surface of the water on the uniform slope EO; that is, the decrease in velocity head accompanying the increase in depth is just sufficient to raise the water along the line EO. The critical conditions of flow exist at the section NP.

requires increasingly greater changes in velocity to supply the required energy, and hence the depth, which varies inversely as the velocity changes more rapidly.

When the water reaches the section NP, the depth must increase at the same rate as the surface rises. Hence at this section the bottom must be level and the depth is the critical depth previously described and shown at NP in figure 1. For $Q = 100$ and $g = 32$, this critical depth, $D_c = 6.79$ feet. As stated before, at this critical point, the velocity head is one-half of D_c.

In order that the surface may rise as the water flows beyond this section, the bottom must descend as indicated in figure 4, which is drawn to scale. If the depth become infinite, so that the entire kinetic energy may be converted into head, the surface may rise to O as a maximum limit.

The bottom need not follow exactly the curve shown. If it follows any smooth curve rising to the same maximum height P,

the water surface will follow a corresponding curve having for each elevation the proper height above the bottom.

A third method might be used to raise the water by converting velocity head into pressure head. If the bottom of the conduit is kept level as in figure 1, but if the sides, instead of remaining parallel to give a uniform width to the channel as in that diagram, are made to approach each other and then diverge, any desired rate of change of the velocity head may be secured.

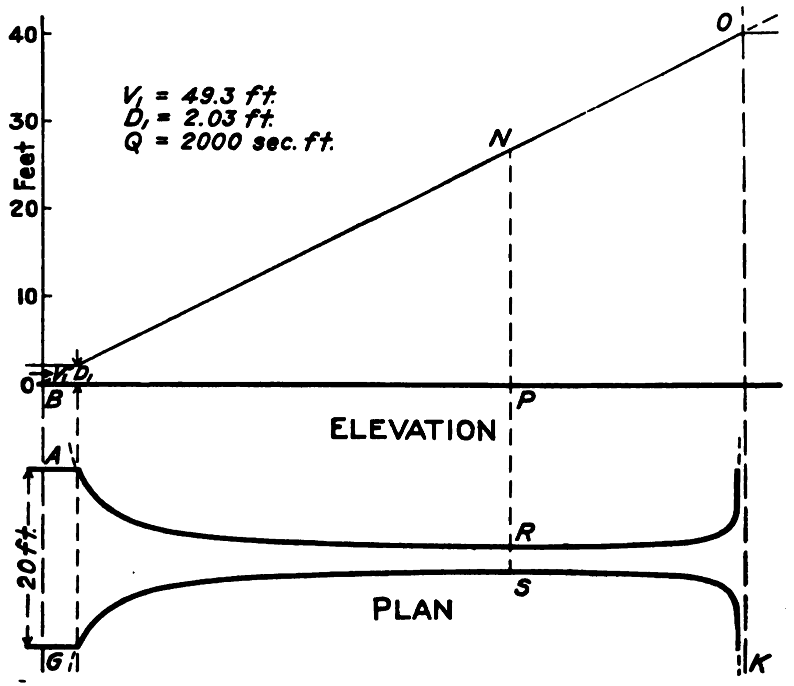


FIG. 5.—DIAGRAM ILLUSTRATING FLOW IN AN OPEN CONDUIT OF VARIABLE WIDTH.

Bottom level, no friction or impact. The variation in width is that necessary to raise the water surface on a uniform slope.

Figure 5 shows the variation in width necessary to secure a uniform slope to the water surface. The minimum width *RS* is only 0.13 of the original width *AG*. At the section of minimum width the depth of water, *NP*, is 26.66 feet and the velocity is 29.2 feet per second. Figure 5 is drawn to show a conduit whose original width *AG* is 20 feet. Otherwise it is on the same scale as figures 1 and 4. The curves showing the variable width, if

prolonged would be asymptotic at the two ends to the straight lines *AG* and *OK*. Any other curves having the same minimum opening *RS* would do as well. The only difference would be that the rising surface of the water would not be a plane but a curved surface.

So far we have been discussing a theoretical condition of flow, having assumed that there is no friction or loss of energy by impact. How far does this represent or approximate conditions as they may exist in actual flow of water? This is an important matter to determine, and fortunately our knowledge of hydraulic phenomena enables us to go a certain distance in answering this question. In the first place, it is to be remembered that of the three distinct methods for converting velocity into head, described above and illustrated in figures 1, 4, and 5, any two, or all three, may operate together without conflict. Surface friction in absorbing energy in one sense is comparable to a rising slope of the conduit bottom, similar to that of the left hand portion of figure 4. Hence, if the amount of friction is known, or is susceptible of estimate, it may be allowed for by the method of combining figure 4 with the other processes.

Impact is not so easily disposed of. In most hydraulic operations it is desired to avoid the occurrence of impact entirely. If it occurs, it is likely to introduce at once an element of extreme variability and uncertainty, not susceptible of accurate prediction or calculation except in certain cases. Hence the present question with us may be considered to be, with how great assurance, or with what certainty, may we assume that impact may be avoided in velocity transformations.

Impact does not occur in a flowing stream in which the velocity is being accelerated, that is, in the condition in which pressure or static head is being converted into velocity. But under the reverse condition of converting velocity into pressure, there seems to be an inherent tendency for impact to occur with a corresponding decadence of kinetic energy into heat. This tendency is often so strong that it is a matter of extreme difficulty to overcome it. Thus, while mathematically velocity and pressure are mutually interchangeable or convertible by a simple relation, in applying this relation, we have to remember that it is another law of nature that the change from pressure to velocity occurs naturally and readily (voluntarily, as it were), while the reverse change from velocity to pressure can only be secured against obstacles, almost as though the water were ani-

mated by a desire to avoid the change, and would avoid it, if any way were open for it to do so.

With this in mind, the methods shown in figures 1, 4, and 5, may be compared. Experience shows that by figure 1, it is comparatively easy to secure the results desired. The water confined on all sides cannot escape the transformation. By figure 4, the change is much more difficult. The water, being open at the top, has a direction of possible motion and a source of disturbance difficult to control. The place of particular danger is at N, the surface of the water when flowing at the critical depth. At this particular place there is no longer any restraint which compels the surface to take any particular direction. It could take any other slope, even a vertical one, so far as the mathematical theory goes. Being, then, in this state of unstable equilibrium, a very slight disturbance may suffice to modify seriously its motion. One result of such a disturbance would be to start a hydraulic jump, to be discussed later, which might end in an entirely new condition of flow.

Figure 5 shows the same condition of instability at the section of critical flow RS. This method would probably be more difficult to adjust even than that of figure 4.

It is interesting to note that if the direction of flow is reversed, so that the velocity is constantly increasing instead of decreasing, all three methods would work equally well, and a choice between them would be indifferent. In fact, any device approaching these curves would yield results following the mathematical theory without danger of impact.

If, on the other hand, the flow is from left to right, as shown in figures 1, 4, and 5, it is impossible for the depth to increase in the absence of friction or impact except by means of some such agency as those illustrated. If the channel is of uniform width, with level bottom, open at the top, and frictionless, it is impossible for the depth of flow to increase in general except by a form of impact which will next be discussed. The exception is the limited amount of possible variation in the immediate vicinity of the critical depth.

SECTION IV.—THE HYDRAULIC JUMP

When a shallow stream moving with a high velocity strikes water of sufficient depth there is commonly produced a striking phenomenon which has been appropriately called the *hydraulic jump*. It consists of an abrupt rise in the surface in the region

of impact between the rapidly moving stream and the more slowly moving wall of water, accompanied by a great tumbling of the commingling water, and the production of a white foamy condition throughout the moving mass. Under suitable conditions this hydraulic jump remains steadily in one position. The surface at the beginning of the abrupt rise is constantly falling against the oncoming stream moving at high velocity, and farther along in the jump, masses of water are continually boiling to the surface from greater depths. So much foam is produced that some time must elapse before it can all rise to the surface and the water again become clear. This phenomenon is constantly illustrated in the surf of the sea-shore.

The correct mathematical theory of the hydraulic jump was apparently first worked out by the Frenchman, Belanger, in 1838. It was copied in Bresse's "Cours de Mécanique Appliquée," published in Paris in 1860; in the article on Hydrodynamics by W. C. Unwin in the ninth edition of the Encyclopedia Britannica about 1880 and in subsequent editions; it is given in the article by A. H. Gibson, entitled "The Formation of Standing Waves in an Open Stream," Paper No. 4081, Minutes of Proceedings of The Institution of Civil Engineers, Vol. CXCVII, Session 1913-1914, Part III; and is quoted in the discussions accompanying the paper of K. R. Kennison, entitled "The Hydraulic Jump in Open Channel Flow at High Velocities," published in Volume LXXX of the Transactions of the American Society of Civil Engineers, 1916.

The theory of the hydraulic jump may be concisely stated as follows:

Let *abfe*, figure 6, represent a mass of water moving through a hydraulic jump. In a short interval of time it is supposed to

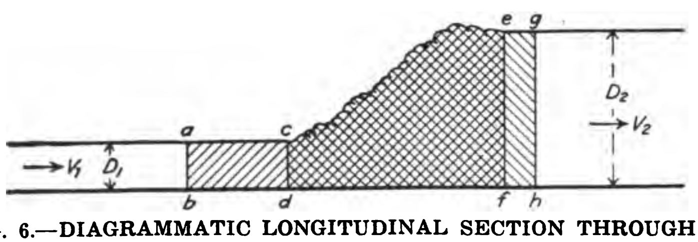


FIG. 6.—DIAGRAMMATIC LONGITUDINAL SECTION THROUGH A HYDRAULIC JUMP

move to the position *cdhg*. The hydraulic jump has the following characteristics:

(a) The water entering at *ab* has a nearly uniform steady high velocity and is transparent.

(b) The water leaving at *gh* has a fairly uniform but relatively low velocity and is transparent.

(c) Between *c* and *e* the surface rises rapidly and is much disturbed by spraying and spattering. Much of the surface water appears to be moving down the slope. The whole mass is so milky as to suggest the presence of much internal impact. The milky condition of the water reduces its specific gravity and accordingly the surface at the top of the rise is above the normal level, but as soon as all the air bubbles reach the surface, so that the water is again transparent, the surface becomes smooth and level.

The moving mass of water loses much momentum in passing from the position *abfe* to the position *cdhg*. According to Newton's Second Law of Motion, the rate of loss of momentum must be equal to the unbalanced force acting on the moving mass to retard its motion.

Against the face *ab* is the static pressure of the water acting towards the right. Opposed to this force are the static pressure acting against the face *ef* and the surface friction along the bottom *bf*. The latter is small and may be neglected.

Let D_1 = depth of stream entering hydraulic jump,
V_1 = velocity of same stream,
D_2 = depth of stream leaving hydraulic jump,
V_2 = velocity of same stream,
D_c = depth of same stream at critical flow,
V_c = velocity at critical flow, $V_c^2 = gD_c$,
Q = quantity in second feet of flow per unit width of stream,

$$Q = V_1D_1 = V_2D_2 = V_cD_c = \sqrt{gD_c^3},$$

$$x = \frac{D_1}{D_c},$$

$$y = \frac{D_2}{D_c},$$

Mass of water flowing per second $= \dfrac{wQ}{g}$,

Change of velocity $= V_1 - V_2$,

$$\text{Change of momentum per second} = \frac{wQ}{g}(V_1 - V_2)$$

$$= \frac{wQ}{g}\left(V_1 - V_1\frac{D_1}{D_2}\right) = \frac{wQV_1}{gD_2}(D_2 - D_1),$$

$$\text{Static pressure acting on face } ab = \frac{wD_1^2}{2},$$

$$\text{Static pressure acting on face } ef = \frac{wD_2^2}{2}.$$

Therefore,

$$\frac{wQV_1}{gD_2}(D_2 - D_1) = \frac{w}{2}(D_2^2 - D_1^2)$$

Dividing both members by $w\ (D_2 - D_1)$,

$$\frac{QV_1}{gD_2} = \frac{D_1 + D_2}{2} \tag{9}$$

from which may be obtained

$$D_2^2 + D_2D_1 = \frac{2QV_1}{g} \tag{10}$$

or, after substituting V_1D_1 for Q,

$$D_2 = -\frac{D_1}{2} \pm \sqrt{\frac{2V_1^2D_1}{g} + \frac{D_1^2}{4}} \tag{11}$$

By substituting Q/D_1 for V_1 in equation 10 we get

$$D_1D_2\frac{(D_1 + D_2)}{2} = \frac{Q^2}{g} = D_c^3 \tag{12}$$

a graph of which is shown in figure 2.

Dividing through by D_c^3, and substituting x and y for their equivalents

$$xy(x + y) = 2 \tag{13}$$

The graph of this equation is shown in figure 3.

Equation 12 shows that if $D_1 = D_c$, D_2 also equals D_c. The equation is symmetrical in D_1 and D_2. If D_1 is less than D_c, D_2 must be correspondingly greater than D_c, and vice versa. There seems to be no physical phenomenon corresponding to a reversal

of the jump in direction. Therefore, there can be no jump unless D_1 is less than D_c, and the jump, when it occurs, always takes place across the critical depth. As already illustrated in figure 4, water can flow at depths less than the critical depth without necessarily forming a jump. This will be more fully discussed later in connection with backwater curves, where the conditions determining the formation of a jump will be fully stated.

The above demonstration rests upon a law of mechanics as well established as any low of nature, as well proved, for example, as the law of gravitation, and there can be no question of the validity of the results. In the hydraulic jump there is a continuous violent inelastic impact by which kinetic energy is converted into heat.

Facing page 82, in the following paper by Riegel and Beebe, is a diagram from which for any values of D_1 and V_1, the values of D_2 and V_2 may be read directly.

SECTION V.—EXPERIMENTAL DATA ON THE HYDRAULIC JUMP

Measurements of the hydraulic jump were made by Bidone about 100 years ago. Similar measurements have also been made by numerous later experimenters including Bazin, Ferriday, Gibson, Riegel and Beebe. These are all described in the following paper by Riegel and Beebe entitled "The Hydraulic Jump as a Means of Dissipating Energy," see page 74.

SECTION VI.—CONDITIONS WITHIN THE JUMP

It has been shown at the beginning of this discussion that, if water is flowing in a smooth uniform rectangular channel with level bottom, it is impossible for the flow to change to a different depth, in accordance with Bernouilli's theorem, except under the critical condition. We have shown subsequently that, under the theory of inelastic impact, apparently the depth can change abruptly from a value less than the critical depth to a particular related value greater than the critical depth. It has furthermore been abundantly proved by experiment and observation that such a phenomenon does actually occur in nature, and measurements of actual velocities and depths follow closely the theoretical relation. The jump thus produced takes place abruptly and occupies relatively but a small space, but the change in the jump from the initial velocity to the final velocity is very

evidently not exactly instantaneous. At points between the cross sections where the initial and final velocities occur, the average velocity must have intermediate values. At first glance this would seem to contravene the mathematical theory of the jump, which seems to preclude any value for the average velocity after the initial value except the calculated final one.

On account of the unsteady turbulence of the water it seems to be impossible to secure by experiment much information regarding the condition throughout the jump. Nevertheless, it seems to be incumbent upon us to attempt some rational explanation of these intermediate conditions or at least to offer some plausible hypothesis to account for them. This may also furnish suggestions of utility in the practical control and use of the jump.

Following the method used previously, let figure 7 represent a longitudinal section of a stream of water passing through a jump. If we assume for simplicity that the surface rises along the straight line *EL*, then at any cross section the depth will be known, and also the average velocity which varies inversely as the depth.

Consider the forces acting on the mass of water between the vertical sections *EA* and *NP*. In passing through this distance the velocity is reduced from V_1 to V. The rate of decrease of momentum requires an unbalanced horizontal force equal to

$$\frac{wQ}{g}(V_1 - V) = \frac{wQ^2}{g}\left(\frac{1}{D_1} - \frac{1}{D}\right) = wD_c^3\left(\frac{1}{D_1} - \frac{1}{D}\right)$$

Neglecting the effect of friction, as before, the only horizontal forces acting on the mass *ENPA* are the static pressure of $wD_1^2/2$ on the face *EA* and the pressure of $wD^2/2$ on the face *NP*.

In order that these forces should suffice to produce the assumed change of momentum the following equation would have to be satisfied:

$$\frac{wD^2}{2} - \frac{wD_1^2}{2} = wD_c^3\left(\frac{1}{D_1} - \frac{1}{D}\right)$$

This is the same equation as used before in establishing the height of the hydraulic jump. The only values of *D* for which this equation can be true are D_1 and D_2. These relations are

further illustrated in figure 7. The static pressure against any vertical cross section is $wD^2/2$. This pressure is represented by a parabola with its vertex at B.

At the cross section EA this pressure is represented by AR, at the cross section NP by PT, and at the cross section LM by MU. The unbalanced force acting against the mass $ENPA$, due to the different static pressures on the faces EA and NP, and

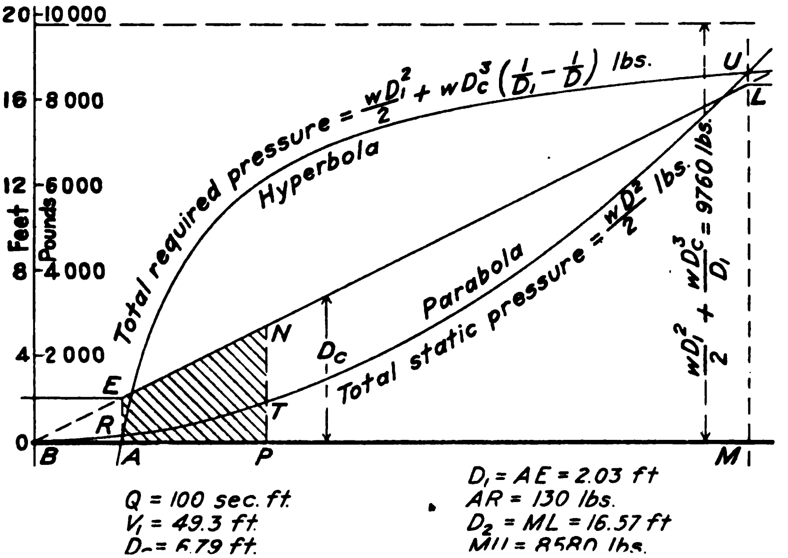


FIG. 7.—DIAGRAM ILLUSTRATING PRESSURE CONDITIONS WITHIN A HYDRAULIC JUMP.

The static pressure against any vertical cross section is given by the parabola $BRTU$; the total pressure required to check the velocity is given by the hyperbola ARU; and the additional pressure required to check the velocity between the points R and U is given by the height of the hyperbola above the parabola, being a maximum at the depth for critical flow.

available to help check the velocity of the water, is represented in figure 7 by the height of the point T above the point R. Between the cross sections EA and LM, this unbalanced force or difference in static pressures is less, as will appear later, than the force required by theory to produce the necessary reduction of velocity at all the intermediate stages. Our problem then is to discover a possible and reasonable source for an additional retarding force to act on the mass $ENPA$ for all positions of the cross section NP intermediate between EA and LM.

The first step is naturally to obtain the magnitude of this

necessary force. The only place where this additional force can act is against the face *NP*. Let *P* equal the total pressure needed by theory against the face *NP*. Then

$$P = \frac{wD_1^2}{2} + wD_c^3\left(\frac{1}{D_1} - \frac{1}{D}\right)$$

Considering *P* and *D* as variables this is the equation of a rectangular hyperbola whose asymptotes are the vertical line through *B* where *D* is zero, and the horizontal line at a height of

$$\frac{wD_1^2}{2} + \frac{wD_c^3}{D_1}$$

above the floor of the channel. This is the value approached by the pressure *P* as *D* becomes infinite. This hyperbola is drawn in figure 7. The curve crosses the parabola previously described at the points *R* and *U*. At all intermediate points the height of the hyperbola above the parabola represents the additional force necessary to produce the requisite decrease in velocity. This excess is greatest at the depth of critical flow.

An inspection of the hydraulic jump in operation suggests the source of this additional force. As a result of the violent impact air in large quantities becomes mixed with the water, giving it a striking milky appearance. Considering this intimate mixture as a homogeneous substance, its density is less than that of water, and since the air displaces some of the water, the surface stands correspondingly higher than it would if no air were present. At such a cross section, then, as *NP* in figure 7, the surface would stand higher than *N*, and the pressure against the cross section would be correspondingly increased.

For simplicity and convenience of discussion let us assume that in any cross section, such as *NP*, the air is mixed uniformly throughout the whole cross section so as to produce a mixture of uniform density. The cross section will then be made up partly of a lace-like mass of water and partly of bubbles of air. If the velocity of the particles of water moving across the section *NP* is the same as before, the total water surface in the cross section is unchanged by the admixture of air. The height of the cross section will be increased in the same proportion as the density is diminished, and the unit pressure on the bottom, the product of the density and height, will therefore remain the same.

Let D represent the depth of any cross section, as NP in figure 7, on the assumption of pure water, and D' the depth at the same cross section on the hypothesis of a uniform mixture of air and water. Then the unit pressure at the bottom of the channel is wD and the total pressure over the cross section would be $wDD'/2$. The height necessary for D', in order to yield the force needed to produce the assumed change of velocity, would be defined by the equation

$$\frac{wDD'}{2} = \frac{wD_1^2}{2} + wD_c^3\left(\frac{1}{D_1} - \frac{1}{D}\right)$$

from which

$$D' = \frac{D_1^2}{D} + \frac{2D_c^3}{D^2D_1}(D - D_1) \qquad (14)$$

If we let

$$\frac{D'}{D_c} = y, \quad \frac{D_1}{D_c} = k, \quad \text{and} \quad \frac{D}{D_c} = x$$

equation 14 becomes

$$y = \frac{1}{x}\left(k^2 + \frac{2}{k} - \frac{2}{x}\right) \qquad (15)$$

This equation is plotted in figure 8 for various values of k ranging from 0.25 to 1.00. Each curve has a maximum value of y given by the relation

$$y_m = \frac{\left(k^2 + \frac{2}{k}\right)^2}{8}$$

and the corresponding value of x is given by

$$x_m = \frac{4}{k^2 + \frac{2}{k}}$$

The point of least density of the mixture of air and water, or the point of greatest admixture of air, is given by

$$x = \frac{3}{k^2 + \frac{2}{k}}$$

from which it follows that this value of x can never exceed unity.

Figure 8 shows that for small values of k the foamy mixture will rise very high into the air. This tends to limit the use of extremely high jumps, by rendering the jump unstable in posi-

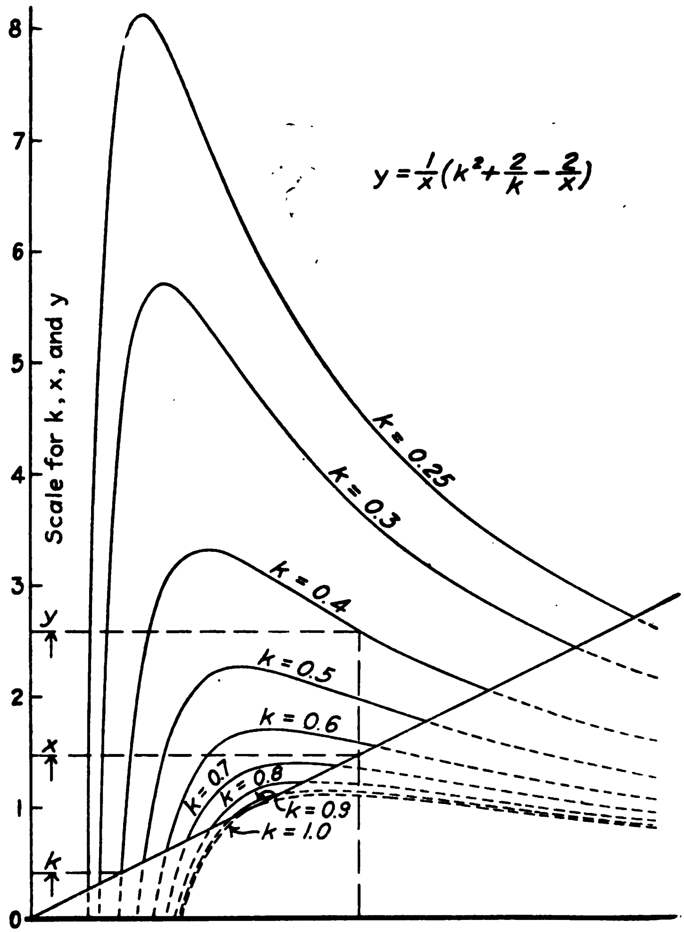


FIG. 8.—DIAGRAM SHOWING IDEAL PROFILES OF THE HYDRAULIC JUMP.

Based on the assumption of a homogeneous mixture of air and water throughout the jump. Scale is such that D_c equals unity.

tion and action. However, in the experiments of Gibson and The Miami Conservancy District described in the succeeding paper, stable jumps were secured when the value of k was as low as 0.25.

SECTION VII.—BACKWATER CURVES

A backwater curve may be defined as the profile of the surface of a stream of water in which all changes of velocity take place in accordance with Bernouilli's theorem. In special cases this curve may become a straight line.

We shall consider only the case of steady flow, that is, that condition in which the flow at any one cross section remains constant in amount, depth, and velocity. For the existence of a backwater curve, except in the special cases above noted, the flow is non-uniform, that is, at different cross sections it varies in depth and correspondingly in velocity.

It is not uncommon in hydraulic design for an occasion to arise requiring the determination of backwater curves, and hence it seems desirable to set forth a method of deriving, explaining, and classifying the different forms which the curves may take. The theory of the backwater curve is sometimes considered impossible to understand; but it is more complex than difficult. By sufficient care in analysis every step can be made perfectly plain. All the mathematics used is simple except in one or two steps, and in these places one entirely unfamiliar with higher mathematics can understand perfectly the scope, meaning, and nature of the successive steps, even if he lacks the mathematical facility required to make them.

In the most general case this subject is much complicated by the large number of independent variables, including the form and dimensions of the channel, the grade of the bottom, the friction, and the quantity of water flowing. In order to adhere to the fundamentals of the subject, we shall consider only a rectangular channel whose width is uniform and so great in comparison with the depth that the hydraulic radius may be considered equal to the depth.

It has been shown in the earlier pages that if such a channel is frictionless and has a level bottom, in general, there can be no variation in depth of flow or in velocity. In other words, there can be no backwater curve; or rather the backwater curve reduces to a simple horizontal straight line, the special case noted above. In order to make the explanation as simple and orderly as possible we shall first consider all the possible cases for a frictionless channel whose bottom is on a grade, and then, finally, we shall introduce the effect of friction.

The notation to be used is important. Numerous quantities must be lettered and given a name. Most of these quantities

reverse in direction in different cases, rendering necessary a careful distinction between positive and negative values. No uniformity exists in the notation used by different authors and apparently every system that can be devised has some objections. We have adopted a notation which, although peculiar in some respects, seems to correspond best with the ordinary language used in describing associated hydraulic phenomena. Since all the mathematical symbols and terms used will be constantly translated into ordinary language in the course of the discussions this correspondence becomes particularly important.

Just as in the first part of this paper the importance of the change from one state to another has been dwelt upon, so here our chief concern will be the process of transition from one condition to another. This process can be studied only by comparing one state with the next one occupied, or by tracing the nature of the difference between one state and another only slightly removed. Two such adjacent conditions are said in mathematical language to be *consecutive*, and to differ by an increment, or sometimes by a differential. It is the province of calculus to engage exclusively in the study of mathematical relations of consecutive states. Hence it seems appropriate to adopt the notation universally used in this branch of mathematics. To those accustomed to the operations of calculus this will seem perfectly natural. To others, the symbols will readily become familiar.* For the present it is sufficient to say that the amount of difference between two consecutive values of any variable is represented by writing d before the letter used for the variable quantity. Thus, y will be used to represent the depth of the water at any place and dy will represent the change of y between two places closely following each other, or, in other words, between two consecutive places.

*Writers of textbooks, in dealing with this or similar subjects, frequently resort to all sorts of awkward and cumbersome systems of lettering, as though they were afraid to use the calculus notation, although they have to use the ideas and explanations which are the fundamentals of the differential calculus. They appear to think that some readers may be frightened away by the calculus symbols, so great a bugbear is this branch of mathematics sometimes considered. The writer has no sympathy with this point of view. With just a little explanation the meaning of the calculus notation can be made perfectly clear. It has become universally adopted in mathematical texts as the most convenient system yet devised. Then why not fearlessly use it in all engineering writings where it is naturally demanded, accompanying it with such preliminary explanation as may seem desirable?

SECTON VIII.—NOTATION

All linear quantities are measured in feet.

Let V = velocity of flowing water, considered as moving in the + direction, from left to right.

Q = volume of water flowing per foot of width of channel. In all the analytical discussion which follows this is understood to be a definite constant quantity.

l = length of channel.

y = depth of water flowing in the channel at any point.

y_c = depth of critical flow as previously explained, that is, $gy_c^3 = Q^2$.

y_n = depth of neutral flow. As will be more fully explained later, this is the depth of flow for a given quantity of water at which the slope of the bottom is equal to the slope required to overcome friction. Water at this depth can flow uniformly for any length with its surface a straight line parallel to the bottom of the channel.

h = elevation of water surface above any convenient datum.

k = velocity head corresponding to V, or $2gk = V^2$.

R = hydraulic radius, and in the present discussion $R = y$.

S = slope of bottom. This will be arbitrarily considered positive for a downward slope towards the right.

S' = slope just sufficient to maintain V and y constant, or the slope required to overcome friction. This may be called the neutral slope.

C = friction coefficient in the Chezy formula $V^2 = C^2RS$.

dl = horizontal distance between two consecutive cross sections whose relations are to be studied.

dy = change in depth between two consecutive cross sections. This will arbitrarily be taken as positive when the depth at the second or right-hand cross section is less than at the other, as shown in figure 9, Case I.

dh = change in elevation of water surface between two consecutive cross sections, considered as positive when surface is falling, figure 9, Case I.

db = change of elevation of bottom between two consecutive cross sections, considered as positive when slope is downward, figure 9, Case I.

dk = change of velocity head between two consecutive cross sections, considered as positive when velocity increases.

df = head required to overcome friction in the distance dl, considered always positive, flow towards right.

SECTION IX.—FLOW WITHOUT FRICTION

If friction is so small that it may be ignored, then, in accordance with Bernouilli's theorem, for every change in surface elevation there must be an equal corresponding change in velocity head, or

$$dh = dk$$

But dk also depends upon the change in velocity, which is dependent in turn upon the change in depth. That is,

$$dk = d\left(\frac{V^2}{2g}\right) \tag{16}$$

and

$$V = \frac{Q}{y} \tag{17}$$

We have already considered to some extent in our discussion of figure 1 the relation between changes in depth of flow and changes in velocity head. In that figure as the depth increases along the straight line, the velocity head decreases as shown by the curve in the upper part of the figure. For a pair of consecutive positions, dy and dk are indicated in figure 1. In accordance with the arbitrary convention adopted above, the negative sign is given to dy because the depth is increasing. The quantity dk is negative because the velocity head is decreasing. In discussing figure 1, it was shown that at the critical depth, when $y = y_c$, dy and dk are exactly equal. When y is greater than y_c, dk is less than dy. As y becomes very large, dk approaches zero in comparison with dy. On the other hand, when y is less than y_c, dk is always greater than dy; and the smaller y becomes, the greater is dk in comparison with dy. All these relations must be kept in mind during the subsequent discussion. The exact relation between dk and dy is given by the equation

$$dk = \left(\frac{y_c}{y}\right)^3 dy \tag{18}$$

This equation is easily derived, by the rules of differential calculus, from equations 16 and 17 above. By using sufficient space it could be here derived independently but this does not

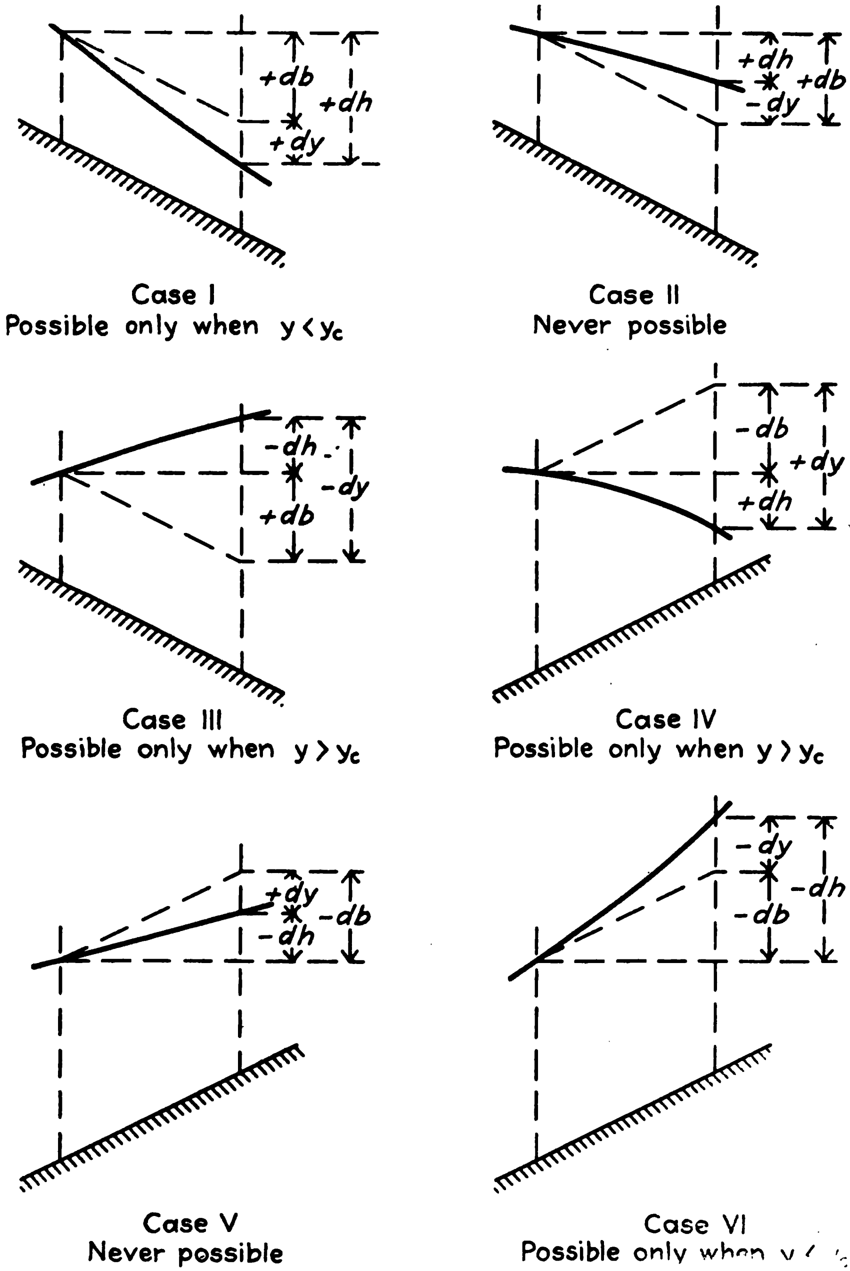


FIG. 9.—ILLUSTRATIONS OF ALL CASES OF FLOW WITHOUT FRICTION.

For each case $dh = db + dy$.

seem necessary, since inspection shows that it agrees with the relation indicated in figure 1.

Figure 9 illustrates six different cases of non-uniform flow without friction that may be conceived as possible and that should, therefore, receive consideration. The full line shows the water surface making a different angle with the stream bed in each case. In the first three cases the slope of the bottom is downward or positive, in the other three the bottom slope is upward. In Case I, the surface falls faster than the bottom. It is convenient to think of this as the typical or normal case. In Case II, the surface falls, but not as fast as the bottom; in Case III, the surface is rising. Cases IV, V, and VI are similarly distinguished.

In figure 9, the + or — sign is shown with each differential to indicate whether the numerical quanity represented is positive or negative. With this aid, inspection shows that the equation

$$dh = db + dy \tag{19}$$

is true for each case.

Remembering the relations, previously discussed, between dk and dy, equation 18, and that $dk = dh$, the following conclusions may be drawn:

Case I is possible only when y is less than y_c, because it is only under these circumstances that dy is less than dk.

Case II is impossible decause dy is negative and dk is positive, and hence they cannot be equal. This might also be explained by saying that if the surface drops, the velocity head must increase; and if the depth, y, increases, the velocity head must decrease; but these conditions are incompatible.

In Case III, the absolute numerical value of dy is greater than that of dh. This can be true only when y is greater than y_c.

Cases I and III, therefore, include all the possible states of flow for a downward slope. If the depth, y, is greater than the critical depth, y_c, Case III obtains. In this case — dy is always greater than dh by the given constant quantity db. But the nearer y approaches to y_c, the nearer, relatively, dy and dh must agree in accordance with equation 18, that is, the nearer the ratio dh/dy approaches to unity. The only way in which both these conditions can be satisfied is for dh and — dy to become larger and larger in proportion to db, which signifies that the surface slope becomes steeper and steeper. As y approaches y_c, the surface slope approaches as a limit a condition of verticality.

When y *is less than* y_c, Case I, the surface slope is downward varying from verticality toward a slope parallel to the bottom as y becomes less and less.

Case IV is possible only when y is greater than y_c because dy is greater than dh.

Case V is impossible because dy and dh have opposite signs, or because the surface cannot be raised and the velocity head increased at the same time.

Case VI is possible only when y is less than y_c, because dh must be greater than dy.

SECTION X.—FLOW WITH FRICTION

To consider the effect of friction, let it be assumed that the coefficient, C, is sensibly constant in the Chezy formula,

$$V^2 = C^2yS \tag{20}$$

for steady uniform flow. For non-uniform flow the above formula is not correct, but we may write

$$V^2 = C^2yS' \tag{21}$$

in which S' is neither the slope of the surface nor of the bottom, but is an ideal slope which, with the given depth and velocity, would overcome friction and maintain a condition of uniform flow.

S' may be called the neutral slope or the friction slope for the given depth and quantity.

Then

$$S' = \frac{V^2}{C^2y} = \frac{Q^2}{C^2y^3} \quad \text{or} \quad y = \sqrt[3]{\frac{Q^2}{C^2S'}} \tag{22}$$

Some value of y will make S' and S equal. Call this y_n. Then y_n is the depth at which the surface would remain parallel to the bottom, or it is the depth at which the bottom slope is equal to the slope required to overcome friction. Hence it may be called the *neutral depth*, for the given bottom slope, S, and

$$S = \frac{V^2}{C^2y_n} = \frac{Q^2}{C^2y_n^3} \quad \text{or} \quad y_n = \sqrt[3]{\frac{Q^2}{C^2S}} \tag{23}$$

It is obvious that when $y_n = y$, the friction slope, S', equals the bottom slope, S; when y_n is greater than y, S' is greater than S; and when y_n is less than y, S' is less than S.

In a short length, dl, the head consumed in overcoming friction, df, will be

$$df = S'dl, \tag{24}$$

and this quantity is always positive.

Since $db = Sdl$, obviously, df will be greater or less than db, according as S' is greater or less than S. Bernouilli's equation now becomes

$$dh = df + dk \tag{25}$$

in which dk, as before, is the change of velocity head and may be either positive or negative.

The six cases of figure 9 might now be reviewed, with the additional complication, that each should be considered under two subdivisions, namely, (a) when S' is less than S, and (b) when S' is greater than S. But some of the cases are impossible and this plan leads to an inconvenient classification. It is found much more satisfactory to take up the various cases according to the following new arrangement. A total of 12 cases will be considered.

First, those in which the channel bottom is sloping downward in the direction of flow, giving 8 cases.

Second, those in which the channel bottom is level, giving 2 cases.

Third, those in which the channel bottom is rising, giving 2 cases.

These groups are further subdivided according to the following scheme:

Channel bottom sloping downward

y_n greater than y_c:

Case A. y greater than y_n.

Case B. y less than y_n but greater than y_c.

Case C. y less than y_c.

y_n less than y_c:

Case D. y greater than y_c.

Case E. y less than y_c but greater than y_n.

Case F. y less than y_n.

y_n equal to y_c:

Case G. y greater than y_c.

Case H. y less than y_c.

Channel bottom level

Case I. y greater than y_c.

Case J. y less than y_c.

Channel bottom sloping upward

Case K. y greater than y_c.

Case L. y less than y_c.

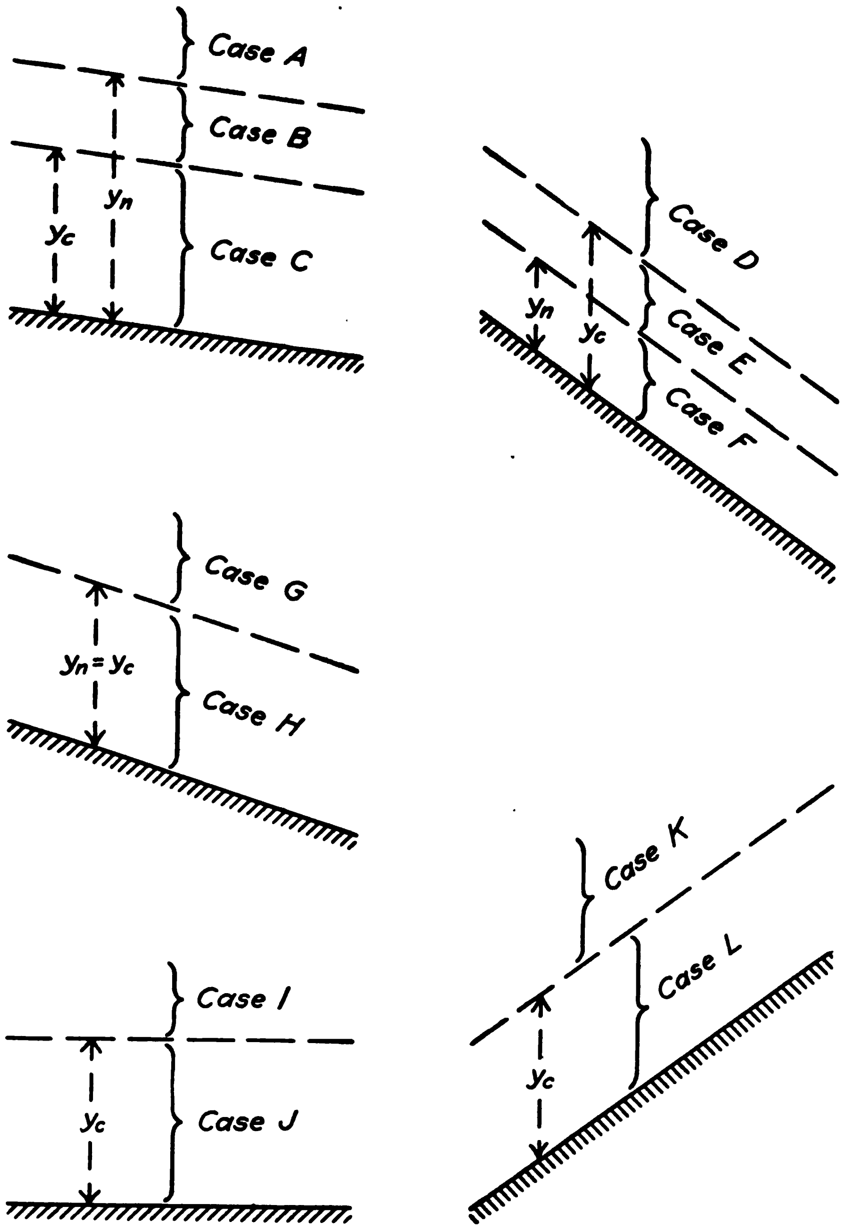


FIG. 10.—CLASSIFICATION OF BACKWATER CURVES.

This scheme of classification is illustrated in figure 10.

For ordinary channels with bottom sloping downward, y_n is greater than y_c, as in the first three cases listed above; but y_n may become less than y_c if the channel is sufficiently smooth or steep.

In accordance with the suggestion previously made, Case B is probably the best to be thought of as the normal or typical case when writing general mathematical expressions and our notation is so chosen that all the quantities are positive in this case. Each case will now be taken up for detailed discussion in consecutive order.

It will be advantageous to bring together all the mathematical relations, previously given, which will be needed in the further discussion.

By geometry

$$dh = db + dy \tag{19}$$

and

$$db = Sdl$$

Hence, by definition of y_n, equation 23,

$$db = \frac{g}{C^2}\frac{y_c^3}{y_n^3}dl \tag{26}$$

By Bernouilli's theorem

$$dh = df + dk \tag{25}$$

hence, from equation 19,

$$db + dy = dh = df + dk$$

By the Chezy formula

$$df = \frac{V^2}{C^2R}dl = \frac{Q^2}{C^2y^3}dl = \frac{g}{C^2}\frac{y_c^3}{y^3}dl \tag{27}$$

Therefore,

$$df = \frac{y_n^3}{y^3}db \qquad \text{or} \qquad \frac{df}{db} = \frac{y_n^3}{y^3} \tag{28}$$

By variation of velocity head, equation 18,

$$dk = \frac{y_c^3}{y^3}dy \qquad \text{or} \qquad \frac{dk}{dy} = \frac{y_c^3}{y^3} \tag{29}$$

It is important to note that dy and dk in any single case have always the same sign, that is, if one of them is negative the other must also be negative, or vice versa.

Throughout the following discussion dl will be considered a constant quantity, having the same value in all cases; y_c and y_n are constant quantities in each case although they vary in the different cases; db will be constant in each case but will have different values in different cases; df, as before stated, will always be positive, that is, in graphical representation it will always be measured downward from the initial elevation.

Case A. Bottom sloping downward, y_n exceeds y_c, and y is greater than each of y_n and y_c, see figure 11.

Since y is greater than y_n, the friction slope, as shown in previous discussions, is less than the slope of the bed of the channel and the head required to overcome friction in any length is less than the fall of the bottom in that length, or df is less than db. This corresponds with equation 28. Also, db is plus in accordance with the fundamental assumption for this case, and df is always positive. Consequently, in figure 11, we may draw AB and CD to represent db and df, respectively. Now, we know,

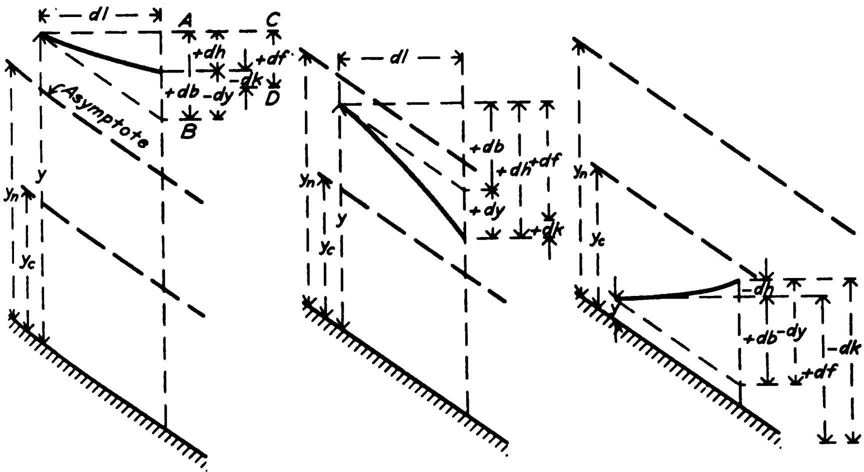


FIG. 11.—CASE A. FIG. 12.—CASE B. FIG. 13.—CASE C.

BACKWATER CURVE RELATIONS WHEN CHANNEL BOTTOM HAS FALLING SLOPE AND y_n EXCEEDS y_c

Fig. 11.—Case A, y is greater than y_n and y_c.
Fig. 12.—Case B, y is between y_n and y_c.
Fig. 13—Case C, y is less than y_n and y_c.

first, that dy and dk must be drawn in the same direction, since they always have the same sign; second, that when drawn from the points B and D, respectively, they must end at the same elevation, see equations 19 and 25. Since y is greater than y_c,

by the previous discussion of the relation between velocity head and depth, dy must be greater than dk. Now, the only way we can draw dy and dk from the points B and D, and make dy greater than dk, is to draw them upward. Hence, dy and dk must be negative, as represented on figure 11.

This same conclusion may be said also to follow algebraically from the equation:

$$db + dy = dh = df + dk$$

For, if db is greater than df, the only way that dy can be numerically greater than dk, having the same sign, is for both dy and dk to be negative. This means that the depth, y, increases in the direction of flow.

Also, df must always be greater than dh, for otherwise dk would be positive and opposite in sign to dy, which is impossible.

As the depth y increases indefinitely toward the right, df will decrease towards zero, and therefore dh, always smaller than df, will likewise approach zero, so that the curve will ultimately approach a horizontal direction.

In the other direction, that is towards the left, since y is always decreasing, it will gradually approach y_n. Similarly, df will approach db. Hence, dk and dy, in order to maintain a ratio different from unity, which is necessary because y is greater than y_c, must become smaller and smaller and must ultimately approach zero. Therefore, dh will approach db, and the curve becomes asymptotic to the straight line parallel to the bed of the channel and a distance y_n above it.

The remaining property of this curve to be proved and the most difficult of all is that dh can never be negative, nor equal to zero, except at an infinite distance toward the right. This will indicate that the curve contains no point of reversal, or that it has no kinks in it.

If dh were zero, db would equal $-dy$, and df would then equal $-dk$. Then it would be true that

$$\frac{df}{db} = \frac{dk}{dy}$$

But equations 28 and 29 show that df/db and dk/dy cannot be equal, except when y is infinitely great, making df and dk both zero. Therefore, dh cannot be zero within a finite distance.

Similarly, if dh were a negative quantity, $-dy$ would be greater than db, and $-dk$ would be greater than df by the same amount. The two fractions

$$\frac{df}{db} \quad \text{and} \quad \frac{dk}{dy}$$

would then have the relation that the numerator and denominator of the second would both be greater than the corresponding terms of the first fraction by the same quantity. In other words, the second fraction would be formed by adding the same quantity, $-dh$, to the numerator and denominator of the first. Now, the fraction df/db, by equation 28, is less than unity. When both terms of such a fraction are increased by the same amount the new fraction must be greater than the original fraction, so in this case dk/dy would have to be greater than df/db. But equations 28 and 29 show that dk/dy is less than df/db. Hence dh cannot be negative, as assumed.

This case is perhaps the form of backwater curve most frequently encountered in engineering practice, and corresponds to the curve created above a dam or other obstruction.

Case B. Bottom sloping downward, y_n exceeds y_c, and y is between them.

Figure 12 illustrates this case. From equation 28 df must be greater than db, while dk is less than dy in accordance with equation 29. Hence, the illustration shows that dk must always be positive. Therefore, dy and dh must also always be positive, and accordingly the depth always decreases toward the right and the surface slopes downward in that direction.

As y approaches y_c, the ratio of dk to dy approaches unity, while at the same time the difference between them has a definite value equal to the difference between df and db. The only way that these two conditions can be satisfied is for dk and dy to become larger and larger in comparison with db. Therefore, the curve becomes steeper and steeper and approaches a condition of verticality as it approaches the critical depth y_c.

Towards the left as y approaches y_n, df will approach db. Therefore, dy and dk, in order to maintain their proper ratio, will both have to approach zero, and the curve will be asymptotic to the straight line parallel to the channel bed and at a distance y_n above it.

This case exists at the end of a flume whose flow is discharging freely into the air, and also just above a sudden drop in the bottom of a channel.

Case C. Bottom sloping downward, y_n exceeds y_c, and y is less than y_n and y_c, see figure 13. Since y is less than y_n, df is greater than db. Since y is less than y_c, dk is greater than dy. Therefore, dk and dy must be negative.

From equations 28 and 29 df/db must be greater than dk/dy. Since both these fractions are greater than unity, the above condition can be true only when dh is negative so that dk shall be greater than df, and dy greater than db, by the same amount, as indicated in figure 13. Therefore, the surface slopes always upward toward the right, and the depth, y, likewise increases in the same direction.

As y approaches y_c, and the ratio dk to dy approaches unity, these two differentials must become larger and larger in comparison with db and df, and therefore the surface becomes continually steeper and approaches a condition of verticality as y approaches y_c.

Towards the left, as y becomes small, df becomes very large. Figure 13 shows that dk must also become very large. Then they may be considered as sensibly equal. Therefore, dividing equation 28 by 29 we get

$$\frac{dy}{db} = \frac{y_n^3}{y_c^3}$$

as the limit approached as y approaches zero. This means that the curve approaches the bottom of the channel at a definite angle, not tangent and not perpendicular, and leads to the very curious result that the backwater curve has a definite length, and is definitely limited at both ends.

Case D. Bottom sloping downward, y_n less than y_c, and y larger than each of y_n and y_c, see figure 14. In this case df is less than db, and dk is less than dy. These conditions cannot both be true unless dy and dk are negative.

Also from equations 28 and 29, dk/dy is greater than df/db. This cannot be true unless dh is negative. Hence the curve rises toward the right, and the water becomes continually deeper towards the right.

As y increases indefinitely, both df and dk must approach zero. Hence dh will also approach zero, and the curve approaches

a horizontal direction as it recedes indefinitely toward the right.

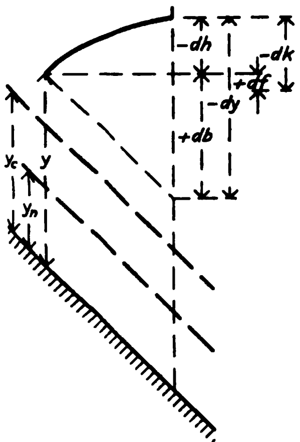


FIG. 14.—CASE D.

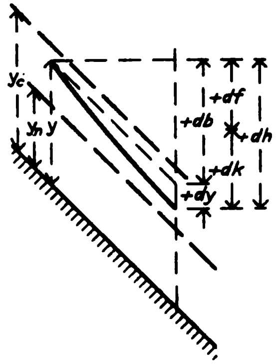


FIG. 15.—CASE E.

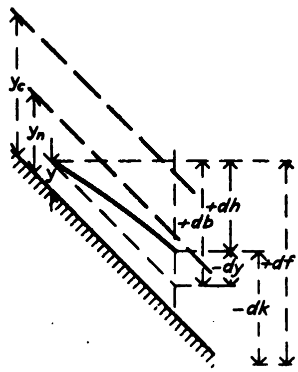


FIG. 16.—CASE F.

BACKWATER CURVE RELATIONS WHEN CHANNEL BOTTOM HAS FALLING SLOPE, AND y_n IS LESS THAN y_c.

Fig. 14.—Case D, y is larger than y_n and y_c.
Fig. 15.—Case E, y is between y_n and y_c.
Fig. 16.—Case F, y is less than y_n and y_c.

In the other direction as y approaches y_c, the ratio of dk to dy approaches unity, but the ratio of df to db does not. The only way in which these conditions can be satisfied is for dk and dy to become greater and greater in comparison with db and df, or, in other words, for the curve to become steeper and steeper and to approach a vertical direction as it approaches the depth y_c.

Case E. Bottom sloping downward, y_n less than y_c, and y between them in value, see figure 15. In this case, df is less than db but dk is greater than dy. Inspection of figure 15 shows that these conditions cannot be true unless dy is positive, and therefore dk and dh are also positive. Hence the surface drops and the depth dy decreases towards the right, as shown in figure 15.

As y decreases toword y_n, df and db must approach equality while dk and dy do not. The only way in which these conditions can be satisfied is for dk and dy both to approach zero. Therefore the curve toward the right is asymptotic to the straight line parallel to the bed of the channel and a distance y_n above it.

In the other direction as y approaches y_c, dk and dy approach equality which requires that they become indefinitely great in comparison with df and db. Hence the backwater curve approaches verticality as it approaches the depth y_c.

Case F. Bottom sloping downward, y_n less than y_c, and y less than each of y_n and y_c, see figure 16. In this case df is greater than db, and also dk is greater than dy. Therefore dy, and consequently also dk, must be negative. Hence the depth increases towards the right.

Also the ratios df/db and dk/dy are each greater than unity, but the latter is the larger. This can only be true if dh is positive, as drawn in figure 16. Hence the surface slope is downward toward the right.

As y increases towards y_n, df will approach db, while dk and dy do not approach equality. Hence dk and dy will each, of necessity, approach zero, and the curve becomes asymptotic to the straight line a distance y_n above the channel bed.

In the other direction, towards the left, as y approaches zero, df and $-dk$ must both become indefinitely great and hence may be considered sensibly equal. Therefore dividing equation 28 by 29

$$\frac{dy}{db} \doteq \frac{y_n^3}{y_c^3}$$

and the backwater curve will intersect the bottom of the channel at a definite acute angle. Hence, like Case C, this curve is definitely limited at both ends and has a definite length.

Case G. Bottom sloping downward, y_n and y_c equal, and y greater than each of them, see figure 17. In this case df is less than db and dk less than dy. By equations 28 and 29 the ratios df/db and dk/dy are equal. The only way in which this can be possible is for db and $-dy$ to be equal and likewise for df and $-dk$ to be equal. This requires that dh be zero and the backwater curve becomes a horizontal straight line, as indicated in figure 17.

Case H. Bottom sloping downward, y_n and y_c equal, and y less than each of them, see figure 18. By the same reasoning as in the preceding case the curve is shown to be a horizontal straight line. This is illustrated in figure 18.

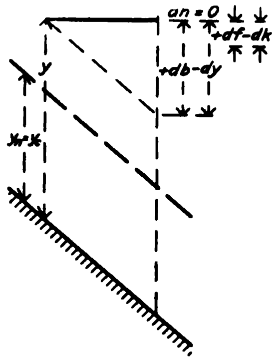


FIG. 17.—CASE G.

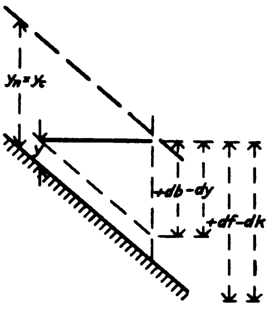


FIG. 18.—CASE H.

BACKWATER CURVE RELATIONS WHEN CHANNEL BOTTOM HAS FALLING SLOPE AND y_n AND y_c ARE EQUAL.

Fig. 17.—Case G, y is greater than y_n and y_c.
Fig. 18.—Case H, y is less than y_n and y_c.

Case I. Bottom level and y greater than y_c, see figure 19. This case and the following one, both with a level channel bottom, correspond closely with Cases B and C, if, in the latter, y_n has become infinite. This is true since as the slope S becomes smaller and smaller y_n becomes indefinitely greater. Or in equation 26, if db be made zero, y_n becomes infinite. Figure 19 shows the differential relations. The detailed discussion is not given on account of its similarity to Case B.

Case J. Bottom level and y less than y_c, see figure 20. This case corresponds to Case C with y_n increased indefinitely. Fig-

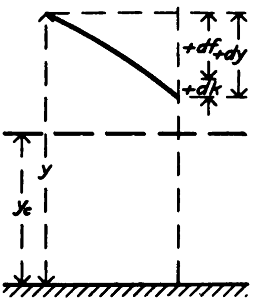


FIG. 19.—CASE I.

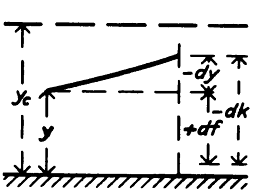


FIG. 20.—CASE J.

BACKWATER CURVE RELATIONS WHEN CHANNEL BOTTOM SLOPE IS ZERO.

Fig. 19.—Case I, y is greater than y_c.
Fig. 20.—Case J, y is less than y_c.

ure 20 shows the differential relations. The detailed discussion is omitted on account of its similarity to Case C.

Case K. Bottom sloping upward and y greater than y_c, see figure 21. In Cases K and L, y_n does not enter into the classification. It has no physical significance, as is indicated by reference to equation 28. When db is negative, as it must be under the scheme of notation, y_n must be negative, and be measured below the bed of the channel.

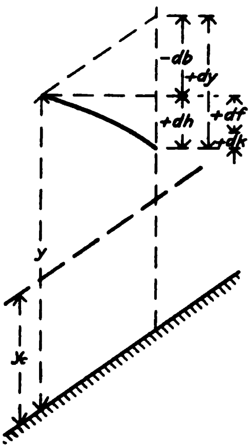


FIG. 21.—CASE K.

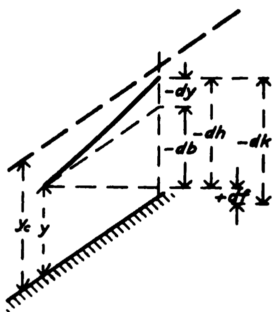


FIG. 22.—CASE L.

BACKWATER CURVE RELATIONS WHEN THE CHANNEL BOTTOM SLOPES UPWARD.

Fig. 21.—Case K, y is greater than y_c.
Fig. 22.—Case L, y is less than y_c.

Since y is greater than y_c, dk is less than dy. Therefore, it is evident that y must decrease, as shown in figure 21. Hence dy, and therefore dk also, is positive. As df is always positive, dh must likewise be positive.

As y approaches y_c, dk and dy approach equality, which they can only do by increasing indefinitely. Hence the curve approaches vertically as it approaches the depth y_c.

In the other direction, towards the left, as y increases steadily, both df and dk approach zero. Therefore, dh approaches zero and the curve gradually becomes nearer horizontal.

Case L. Bottom sloping upward and y less than y_c, see figure 22. Since y is less than y_c, dk is greater than dy. Therefore, figure 22 shows that dy must be negative, and the depth y is

increasing toward the right. Similarly, *dh* is negative and the surface is rising toward the right.

As y approaches y_c, dy and dk approach equality, hence they must increase indefinitely in comparison with db and df, and the curve approaches verticality as it approaches the depth y_c.

In the other direction, as y approaches zero, the backwater curve intersects the bottom within a finite distance as in the other cases previously discussed, wherein y is less than y_n and y_c. This may be shown by the following procedure. If we assume for the moment y_n to stand for the depth at which the given quantity of water would flow uniformly in the reverse direction, equations 28 and 29 will be true as before. As y approaches zero towards the left, df and dk must both become indefinitely great, and their ratio will approach unity. Dividing equation 28 by 29, there will then be obtained

$$\frac{dy}{db} = \frac{y_n^3}{y_c^3}$$

Hence, for any finite value of y_n, whether larger or smaller than y_c, the backwater curve will intersect the bottom at a definite acute angle.

The mathematician would treat by means of general equations the 12 cases discussed above separately. Thus, the value of db given by equation 26, substituted in 19, gives

$$dh = \frac{g}{C^2}\frac{y_c^3}{y_n^3}dl + dy \tag{30}$$

The value of df given by 27 and the value of dk from 29, substituted in 25, give

$$dh = \frac{g}{C^2}\frac{y_c^3}{y^3}dl + \frac{y_c^3}{y^3}dy \tag{31}$$

If from this pair of simultaneous equations dy is eliminated, there is obtained

$$dh = \frac{g}{C^2}\,\frac{1 - \dfrac{y_c^3}{y_n^3}}{\dfrac{y^3}{y_c^3} - 1}\,dl \tag{32}$$

If 30 and 31 are equated to eliminate dh, the resulting equation may be written in two ways

$$dy = \frac{g}{C^2} \frac{1 - \dfrac{y^3}{y_n^3}}{\dfrac{y^3}{y_c^3} - 1} dl \tag{33}$$

or

$$dl = \frac{C^2}{g} \frac{\dfrac{y^3}{y_c^3} - 1}{1 - \dfrac{y^3}{y_n^3}} dy \tag{34}$$

From 32 and 33 all the properties of the 12 cases deduced above can be derived by making proper allowances for positive and negative signs.

It is the province of integral calculus to determine, from differential equations like 34, the equations of the curves which have such differential properties. First changing the sign of dy so as to conform to the customary conventions used in calculus and then using standard methods of integration, after a somewhat lengthy procedure, the general integral is obtained in the following form, in which z stands for y/y_n:

$$l = \frac{y_n z}{S} - y_n\left(\frac{1}{S} - \frac{C^2}{g}\right)\left[\frac{1}{6}\log_e\frac{z^2+z+1}{(z-1)^2} - \frac{1}{\sqrt{3}}\operatorname{cotan}^{-1}\frac{2z+1}{\sqrt{3}}\right] \tag{35}$$

Tables of the complicated function contained in the brackets have been computed by Bresse for positive values of z and are published in most textbooks on hydraulics. This covers the first six cases previously described, Cases A to F, inclusive. By the aid of such tables, figures 23 to 28 have been drawn accurately, but with greatly exaggerated vertical scale, showing the shape of the typical backwater curve for each case.

It should be noted that no backwater curve can cross the critical depth and that in several cases the curve has either a beginning or an abrupt ending at this depth. In plotting the curves by the use of Bresse's tables, Cases B and C will appear as one continuous curve, and similarly with Cases D and E. This is because mathematically they are represented by the same con-

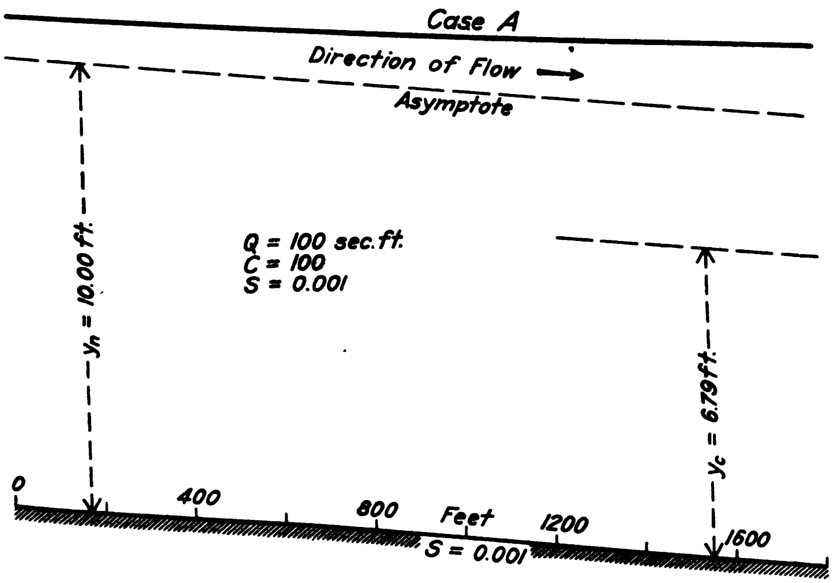


FIG. 23.—TYPICAL BACKWATER CURVE FOR CASE A.

Plotted from Bresse's table of the backwater function. Vertical scale 100 times horizontal scale.

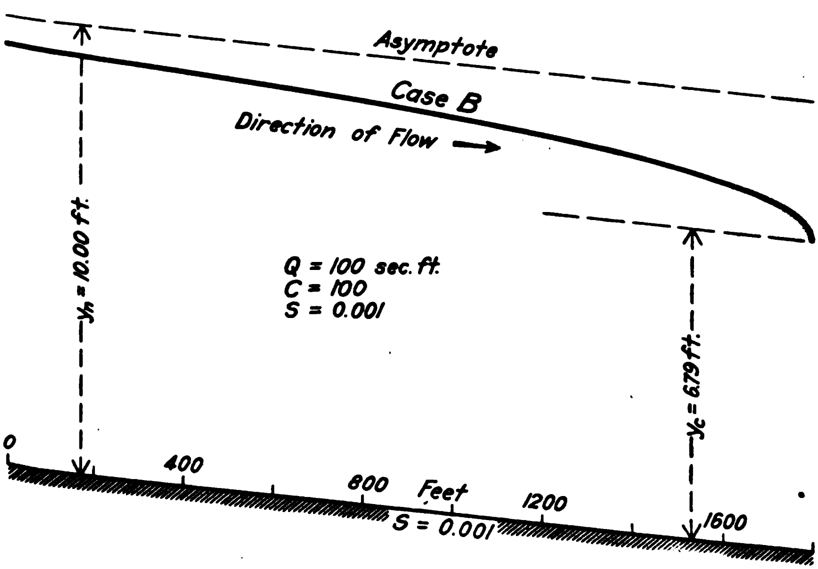


FIG. 24.—TYPICAL BACKWATER CURVE FOR CASE B.

Plotted from Bresse's table of the backwater function. Vertical scale 100 times horizontal.

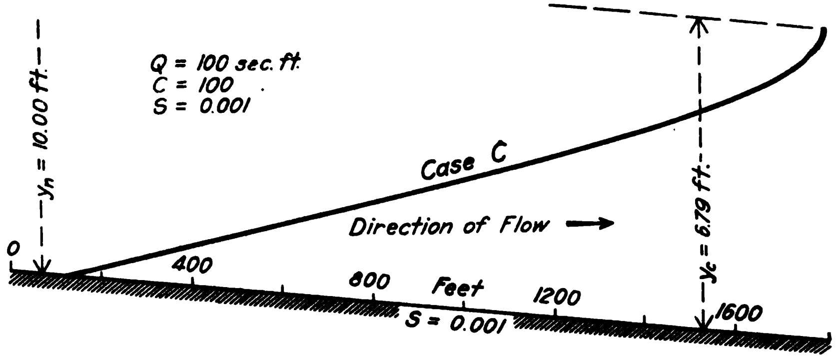


FIG. 25.—TYPICAL BACKWATER CURVE FOR CASE C.

Plotted from Bresse's table of the backwater function. Vertical scale 100 times horizontal scale.

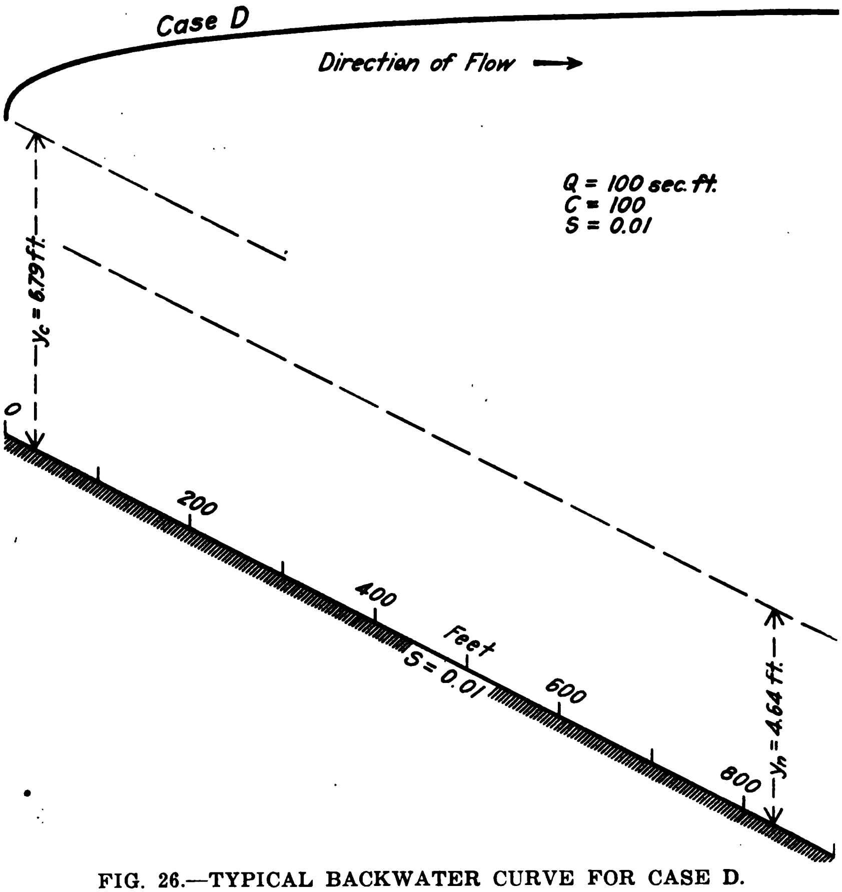


FIG. 26.—TYPICAL BACKWATER CURVE FOR CASE D.

Plotted from Bresse's table of the backwater function. Vertical scale 50 times horizontal scale.

tinuous function. This may be a source of danger for the unwary but the difficulty can be avoided by remembering that no curve under investigation can cross the critical depth. Any point obtained by computation from Bresse's tables, which seems to violate this rule, should be rejected as being beyond the physical limits of the curve under examination.

Case A, figure 23, illustrates the sort of curve produced when the lower end of a long flume is submerged in a reservoir to a greater depth than the neutral or natural depth of flow in the flume.

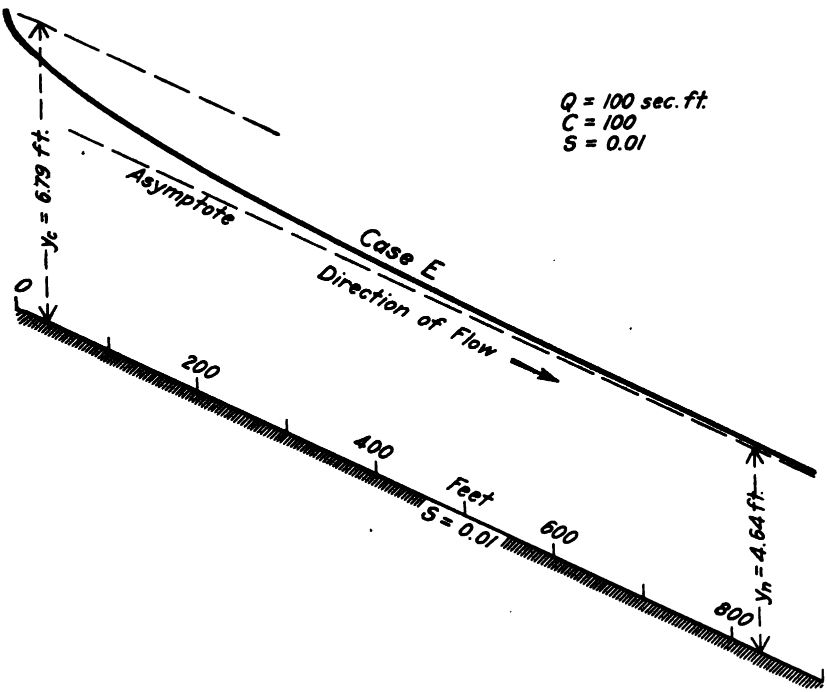


FIG. 27.—TYPICAL BACKWATER CURVE FOR CASE E.

Plotted from Bresse's table of the backwater function. Vertical scale 50 times horizontal scale.

Case B, figure 24, similarly represents the condition when the bottom of the flume at its lower end is submerged in a reservoir to a depth less than the neutral depth. If the depth of submergence is greater than the critical depth, then as much of the curve for Case B will form as lies above the water surface in the reservoir. If the amount of submergence is less than the critical depth the backwater curve of Case B will end abruptly

with its lower end tangent to a vertical line and at a height equal to the critical depth. Below this level there will be a free vertical fall. These last conditions are only approximately realized in an actual case, for the reason that in deducing the theoretical curve vertical components of velocity were ignored.

Case C, figure 25, is very peculiar in that the curve at its lower left-hand end starts from the bottom of the channel at an acute angle and terminates abruptly at its upper right-hand end tangent to a vertical line. Thus, the curve is restricted in length to definite limits in both directions. On this account the curve can exist only under favoring circumstances.

For example, if water issued at high velocity from a reservoir through a submerged gate opening, it could follow this curve, provided that the water could be conducted away by some change in channel conditions, before the right-hand end of the curve should be reached. Such a change might be a sufficient increase in the grade of the channel. The higher the initial velocity of the issuing water, the farther down on the curve towards the left it would begin. At the end of the curve, where it intersects the bottom of the channel, the velocity of the water would be infinite, so this point represents a limit that could never be reached physically.

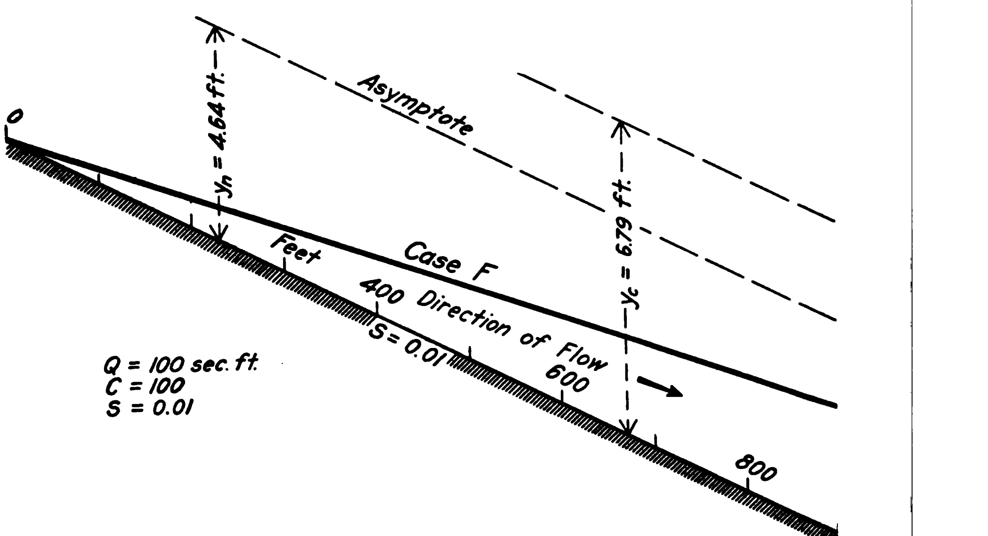


FIG. 28.—TYPICAL BACKWATER CURVE FOR CASE F.

Plotted from Bresse's table of the backwater function. Vertical scale 50 times horizontal scale.

Cases D, E, and F, as shown in figures 26, 27, and 28, correspond somewhat to Cases A, B, and C, respectively. It will be noticed, however, that some of the slopes and directions are reversed. Cases A, B, and C change into D, E, and F when the grade is increased until the neutral depth is less than the critical depth. The same table from which Case A is calculated gives Cases D and E. Similarly Cases B, C, and F are obtained from the same table.

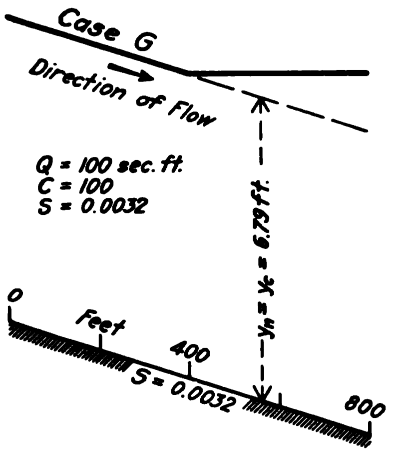


FIG. 29.—TYPICAL BACKWATER CURVE FOR CASE G.

Vertical scale 100 times horizontal scale

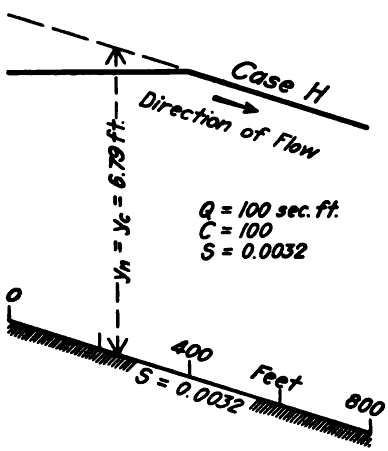


FIG. 30.—TYPICAL BACKWATER CURVE FOR CASE H.

Vertical scale 100 times horizontal scale.

Cases G and H, figures 29 and 30, formed of straight lines, represent the limiting conditions between Cases A, B, and C on the one hand, and Cases D, E, and F on the other. The condition necessary for Cases H and G is that the slope be just sufficient to compensate for friction when the neutral depth and critical depth become equal. On account of the horizontal straight lines for the surface curve, in these cases, they possess manifest advantages in practical use where they can be secured.

Cases I and J, figures 31 and 32, represent the limiting shapes approached by Cases B and C, as the slope of the channel bottom approaches a true level. Under these circumstances the neutral depth, y_n, has become infinite. Hence, equation 34 simplifies to the form

$$dl = \frac{C^2}{g} \frac{y_c^3 - y^3}{y_c^3} dy \qquad (36)$$

The general integral of this is

$$l = \frac{C^2}{g}\left(y - \frac{y^4}{4y_c^3}\right)$$

from which the curves shown in figures 31 and 32, corresponding to Cases I and J, have been computed.

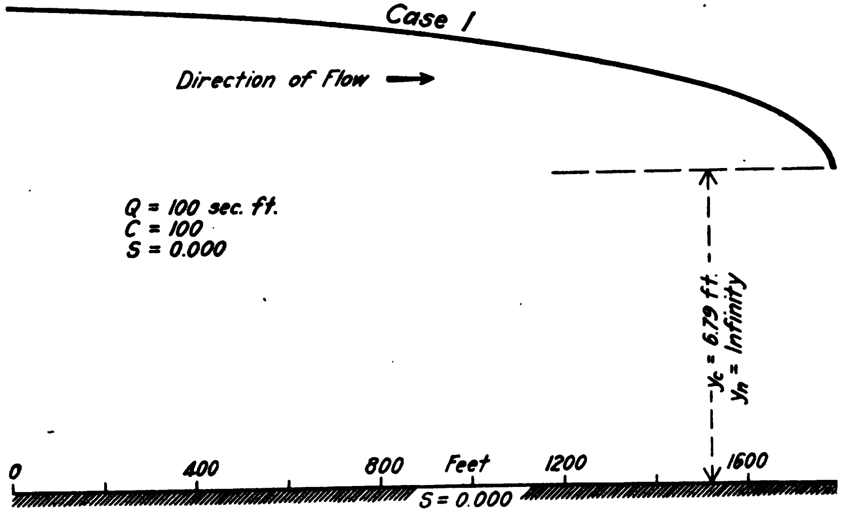


FIG. 31.—TYPICAL BACKWATER CURVE FOR CASE I.
Plotted from equation 37. Vertical scale 100 times horizontal scale.

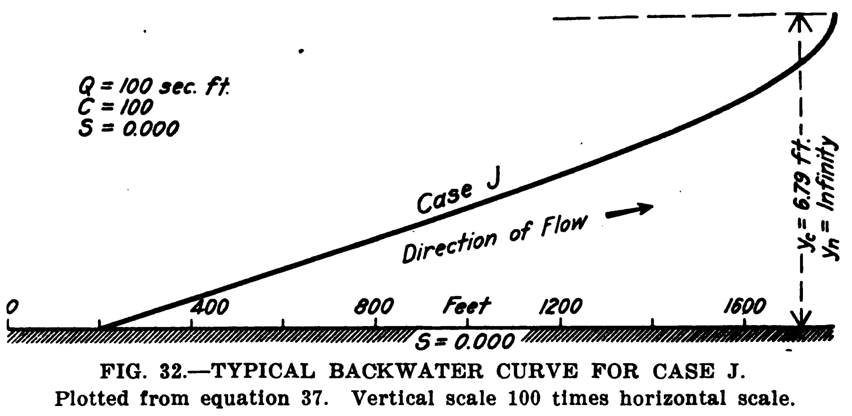


FIG. 32.—TYPICAL BACKWATER CURVE FOR CASE J.
Plotted from equation 37. Vertical scale 100 times horizontal scale.

When the bottom slopes upward, the slope may be considered negative, y_n becomes negative, and hence z is negative. From equation 35, values of the integral corresponding to negative values of z have been computed, and the curves corresponding to Cases K and L, have been drawn, shown in figures 33 and 34.

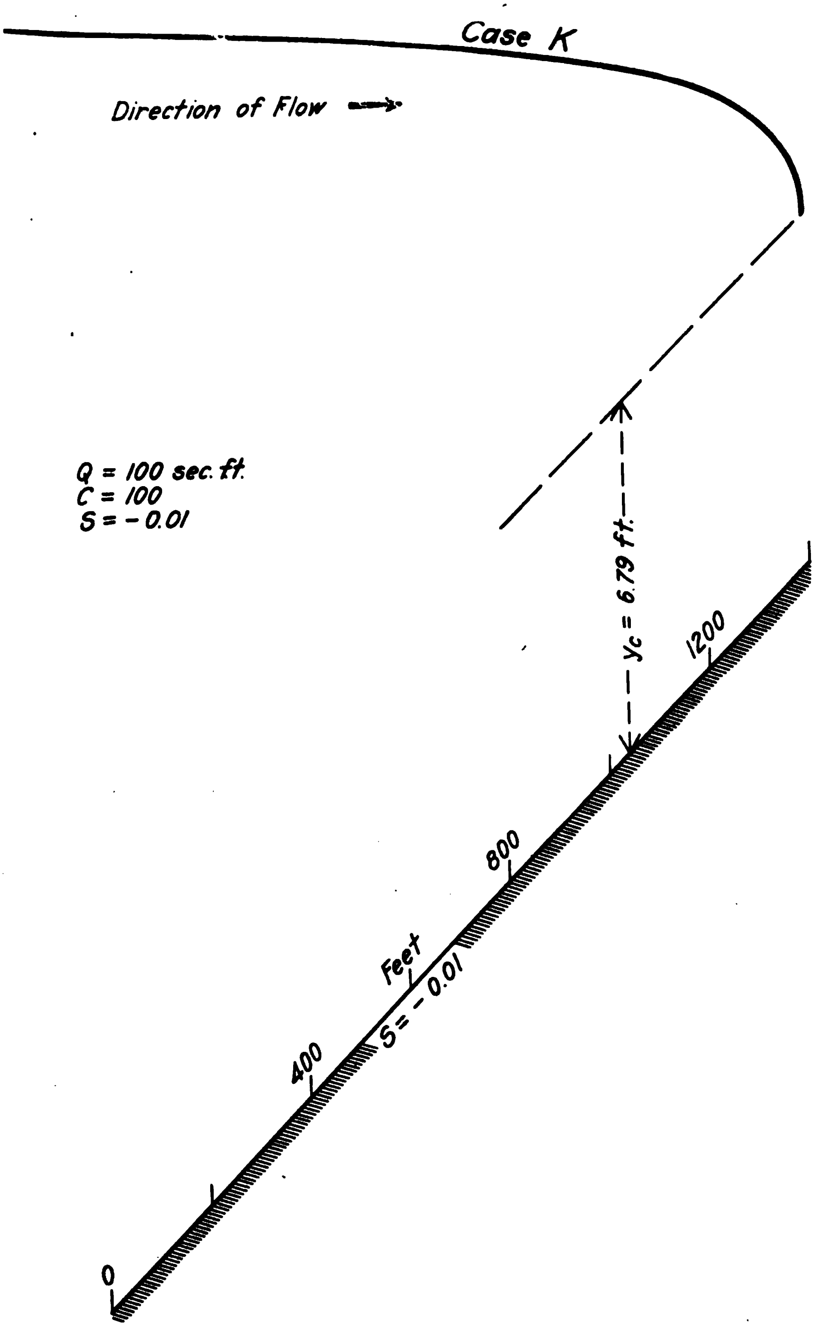


FIG. 33—TYPICAL BACKWATER CURVE FOR CASE K.
Plotted from equation 35. Vertical scale 100 times horizontal scale.

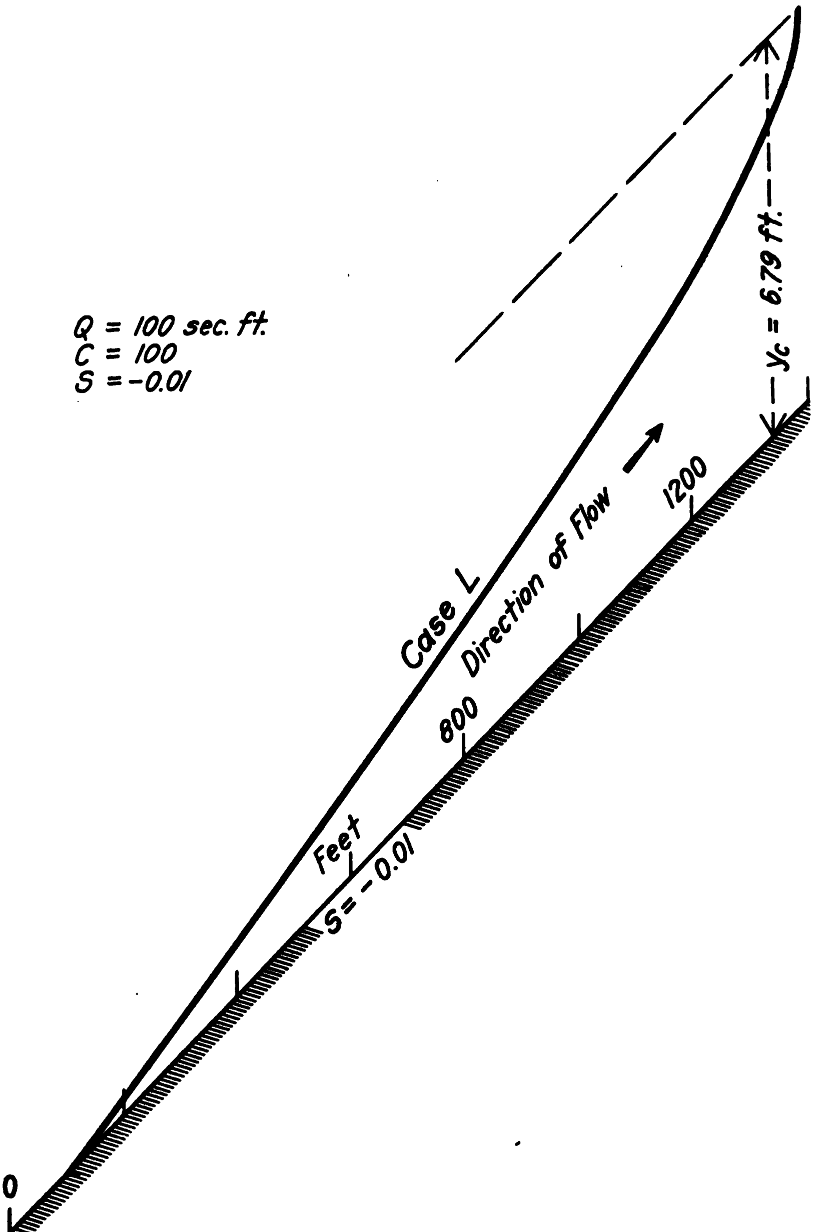


FIG. 34.—TYPICAL BACKWATER CURVE FOR CASE L.

Plotted from equation 35. Vertical scale 100 times horizontal scale.

SECTION XI.—PRACTICAL APPLICATION OF BACKWATER CURVES AND THE HYDRAULIC JUMP

The cases in actual construction in which backwater curves and the hydraulic jump may be of practical importance are too numerous for comprehensive discussion here, but a few typical applications will be mentioned.

If the conditions of flow are known at a particular place in a given open channel, the type of backwater curve existing at that place is easily determined. The critical depth can be computed from the known quantity of water flowing; and the neutral depth from the quantity of flow, slope of the channel, and its roughness. A comparison of the critical depth, neutral depth, and actual depth will determine at once which one of the 12 possible cases is obtaining at the given place.

It should be stated here that if it is desired to apply the backwater theory to a channel of trapezoidal cross section or to a natural stream channel, the width of channel used should be the width of the water surface, and the depth of flow used should be the average depth obtained by dividing the area of the cross section by the width at the water surface.

There are many possible cases of transition backwater curves caused by changes in the slope of the bottom, shape, and dimensions of the channel, or roughness of the channel. Figure 35

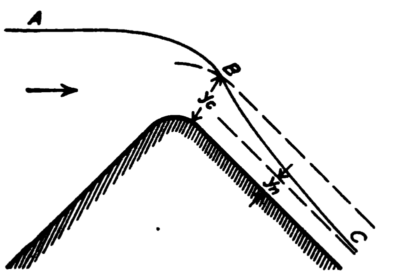


FIG. 35.—TRANSITION BACKWATER CURVES IN FLOW OVER A DAM.

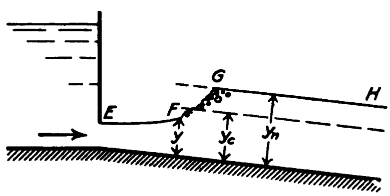


FIG. 36.—CONDITION CAUSING THE FORMATION OF A HYDRAULIC JUMP.

represents flow over a round crested dam with sloping sides. From *A* to *B* the backwater curve has the general form of Cases K, I, and B, and is a transition curve formed by combining these cases. At some point, as *B*, the neutral depth and the critical depths are equal and the actual depth crosses the critical depth line. Below this point the curve *BC* has the general form

of Case E, and is concave upward, while above the point B the curve is convex upward.

Figure 36 represents an important case, one in which water emerges from an orifice or sluiceway under a head and hence with a relatively high velocity of efflux. If the slope of the outfall channel is slight, so that y_n is greater than y_c, while the depth of efflux is less than y_c, then if the velocity of efflux is sufficiently great a hydraulic jump is bound to occur at some point in the outfall channel. From the point of efflux E, the backwater curve EF will be of Case C. When the depth has increased sufficiently so that a hydraulic jump will just carry the depth across the critical depth to the neutral depth y_n, the jump will occur. With a given critical depth y_c and a definite final depth y_n, the initial depth y, for a hydraulic jump is definitely determined as shown in the previous discussion of the equation for the hydraulic jump. This determines the location of the jump.

In case the depth at the orifice should be greater than y, no jump could occur. Instead, the outfall channel would fill to the depth y_n entirely to its upper end and the orifice would become submerged. The only way in which the depth in a case like that shown in figure 36 can increase to a value greater than the critical depth is through a hydraulic jump. In some circumstances this might be a source of danger.

In general, flow at the critical depth may be considered as a possible source of danger, and hence worthy of especial consideration. With an increasing velocity, the critical depth may be passed through, as in figure 35, without harm. With a decreasing velocity, however, such as exists in figure 36, the critical depth cannot be passed through without the heavy internal disturbance accompanying the hydraulic jump, except in the cases when the neutral depth and critical depth coincide as represented in Cases G and H, figures 29 and 30.

The above discussion may serve to indicate the method of analyzing other transitional backwater curves that may be encountered in practice.

THE HYDRAULIC JUMP AS A MEANS OF DISSIPATING ENERGY

SECTION XII.—INTRODUCTORY

The use of retarding basins in the plans of the Miami Conservancy District involves the temporary storage of a large portion of the flood runoff and the gradual discharge of the entire runoff through restricted outlets in the dams. The water discharges at great velocity when the level in the basins is high, and of course, has tremendous kinetic energy. With a flood similar to that of 1913, the discharge at the several dams would vary from 9,000 to 50,000 cubic feet per second and the velocities in the conduits would be from 50 to 60 feet per second. To release water at such velocity into river channels of unconsolidated materials might jeopardize the safety of the dams, since the amount and extent of the erosion that would result could not be predicted. It was regarded, therefore, as essential to secure so low an effluent velocity that no erosion need be feared in the channel below. In other words, the bulk of the energy must be dissipated before the water could be carried in a natural channel.

Investigation showed that the same problem had been presented before, although under different circumstances. For instance, the Catskill Aqueduct of the New York City water supply has a blow-off or by-pass channel into Croton Lake, where provision is made for a discharge of 1,500 cubic feet per second at a velocity of 70 feet per second, the means employed being a large spraying device. At the Waldeck Dam* in Germany a grating of vertical I-beams is placed in the path of the discharging water, the maximum flow in this case being 9,000 second feet at 65 feet per second velocity. At the Gatun and Bassano Dams, piers were placed at the foot of ogee spillways to effect a dissipation of energy. However, these measures were either ineffective or inapplicable to the local conditions.

In view of the lack of large scale precedents and the importance of insuring the dissipation of energy, it was decided to make an experimental investigation to determine the most satisfactory type of structure. Beginning with a model of an outlet channel provided with baffle piers somewhat resembling

*See Journal, Engineers Society of Pennsylvania, May, 1916.

those at Gatun spillway, investigations were made of a number of schemes, the plant being modified from time to time. The hydraulic jump was finally developed as the most practicable means of dissipating the energy, and a form of channel was devised which would insure its occurrence. The methods pursued and the conclusions reached are described below.

Inasmuch as the studies were made upon water in a state of great turbulence, quantitative measurements were limited in their scope, and the facts and conclusions reached are chiefly qualitative in nature. The fact that the experiments were to be used to determine the design of full sized structures necessitated care in making deductions since slight peculiarities in the model might correspond to great irregularities in the large structure. Observations of tendencies were therefore of great importance, and from them important conclusions were drawn.

The necessity for careful observation coupled with the obscurity of some of the phenomena observed, led to a lengthy study. Precautions were taken to check observations carefully, notably by varying discharge and other conditions when studying any particular type, in order to verify the persistence of phenomena. Field work on the experimental plant covered about five months in the aggregate, and an equal amount of time was consumed in studying and digesting the data. It was felt that the importance of the question to be solved justified considerable expenditure. The results obtained were deemed conclusive for the purposes of the District, and of such general interest to the engineering profession as to justify their publication.

Acknowledgment is due to Edward A. Deeds, President of the Board of Directors of the Miami Conservatory District, who provided the site and most of the equipment of the experimental plant. The experiments of Professor A. H. Gibson, Ferriday, and others, as presented in the discussions of K. R. Kennison's paper on* The Hydraulic Jump in Open Channels at High Velocities were of much value in studying the jump.

The work was guided by the advice and suggestions of D. W. Mead, John W. Alvord, H. M. Chittenden, and S. M. Woodward, Consulting Engineers. Mr. Woodward was in constant touch with the investigations and has studied the hydraulic jump from the theoretic standpoint in the discussion published herewith. Suggestions were received from Chas. H. Paul and W. M. Smith, as well as from other members of the staff, while the

*Transactions, American Society of Civil Engineers, Vol. 80.

work was directed in general by Chief Engineer Arthur E. Morgan. Professor Chas. I. Corp prepared the original designs for the plant during the summer of 1915, but was unable to perform the field work of investigation. John C. Beebe had charge of the studies during the fall of 1915 and the succeeding winter, while R. M. Riegel took up the work in the summer of 1916, and has written this report. Both men were assisted by Myron Cornish.

SECTION XIII.—ORDER OF EXPERIMENTS

The general object of the studies has been stated. More definitely, it was to devise a type of outlet structure whose discharge would be symmetrical and free from erosive velocities, on the sides and bottom of the channel. In general, three methods of attainment were recognized as possibilities.

(a) The water might be caused to rise into the air in some form of jet or sheet, losing its energy in so doing, and subsequently falling into a pool below, whence it could flow away quietly.

(b) The velocity might be diminished by impact upon piers or other baffling devices.

(c) The energy of the water might be dissipated in eddies, impact, and friction in the water itself.

In other words, dissipation of energy might occur through air friction, channel friction, or internal friction. Of course, combinations of any or all of these methods mght be used.

The studies were begun in September 1915, with a channel enlarging in a series of offsets, isolated piers in the form of buttresses being built in it. The form of the channel and the shape and arrangement of the piers were varied but the device as a whole was not satisfactory. The results of the studies of piers are given in Section XVIII, together with the reasons for their rejection.

Devices for spraying water into the air were studied, including various forms of weirs, deflecting piers, and channels, with interesting but unsatisfactory results; see also Sections XIX and XX. The form of the channel was also modified from time to time.

During the manipulation of the model for the above studies, it was observed, partly by accident, that under certain circumstances a hydraulic jump was formed, which apparently effected a considerable dissipation of energy. This led to an investiga-

tion of the jump, and, inasmuch as a conflict in published theories and formulas was found, observations were taken to establish the law controlling it. Additional investigations were made of the effect of a roughened floor upon the position and behavior of the jump. The experiments were deemed conclusive enough to warrant the adoption of the jump as the most practicable method of dissipating the energy of swiftly flowing water.

In the summer of 1916, the studies were resumed, with the object of ascertaining the form of channel which would secure the formation of the jump in the most certain and economical manner. This work dealt but little with theoretic relationships and was directed chiefly to tendencies to depart from the theoretic behavior and to means of counteracting them. Several forms of channel were used, with and without regulating weirs and baffles, and a satisfactory form of channel was developed.

SECTION XIV.—CONCLUSIONS

Experimental results from small scale models must be applied with caution to the design of full-sized structures, and the conclusions herein given must, therefore, be largely qualitative in character, showing tendencies rather than exact ratios or formulas to be rigidly adhered to in design. This principle underlies the following conclusions:

1. When baffle piers are used in a spreading channel below the outlet, the flow is not stable under moderate changes in channel conditions such as variations in discharge and tail water level as well as in the form of the structures. Hence, the analogy between the model and the full-sized structure is inconclusive and cannot be relied upon. Moreover, the impact of great quantities of water upon piers supported by the foundations available in the Miami Valley would involve great expense in order to secure stability.

2. The method of causing the water to discharge into a large pool which submerges the conduit outlet is not successful in preventing high velocities below. The high velocity of the water tends to persist through the pool for long distances below the outlet, and the submerged moving stream is also variable in position.

3. The use of devices to spray the water into the air is effective on the small scale of the model, but costly pool construction is necessary on the large scale to receive the falling

water, and doubt was felt as to the advisability of raising so much water into the air in ignorance of the laws of air friction. Provision against damage by splashing and spreading under heavy discharge would also be very expensive.

4. A device to eliminate energy by so forming the outlet channel as to reverse some of the current and to form horizontal eddies was rejected because the divided currents were unstable under various tail water conditions, and because the effluent tended to be concentrated at the sides of the outlet channel. Its cost on the large scale would also be excessive.

5. For large structures, the hydraulic jump is the most practicable method of securing the desired elimination of energy. Its certainty to occur is demonstrated by observations below many high spillway dams, and the manner of securing it at conduit mouths is established by these experiments. Its use is economical and safe, and, since it is governed by a known theoretic law, its position and magnitude are capable of fairly definite calculation.

6. To secure a stable and uniform jump, the water entering it should be in the form of a sheet of uniform thickness and velocity across the channel. This condition can be secured in the channel below a conduit by providing a smooth and gradual expansion in the sides of the channel, so shaped as to insure continuous contact between the spreading water and the sides. The sides should be tangent to the conduit walls and should not be concave toward the water until the jump is passed. The bottom of the channel should be gradually depressed below the outlet, so that at the point where the jump is desired, there shall be sufficient depth of tail water to produce it.

7. Where, as in the case of The Miami Conservancy District dams, the occurrence of the jump is desired through a wide range of discharge and tail water conditions, the position of the jump can be confined within narrow limits by causing it to occur on a floor sloping downward from the conduit outlets. This may be steepened indefinitely, provided that it does not lie below the parabolic path of the water issuing as a jet from the conduit opening, and provided further that sufficient length of channel be arranged to permit the water to spread into a thin and nearly uniform sheet above the region of the jump.

8. When more than one conduit discharges into the same outlet channel, as is uniformly the case in the Conservancy dams, a concentration of flow is produced at the junction of the spread-

ing water from the conduits. Inequality in discharge from several conduits produces the same effect. In either case there is an irregular jump, accompanied by a concentration of flow in some part of the channel below with erosive velocities. This concentration is prevented by the use of one or more submerged weirs below the jump, which baffle and distribute the concentrations.

9. When the flow is equal in the several conduits, the submerged or baffling weir cannot be below a certain minimum height, depending upon the degree of irregularity in flow above the jump, but weirs of varying height above this minimum are equally satisfactory. When the flow from the conduits is decidedly unequal, however, as may occur when one is obstructed temporarily, a high weir becomes necessary, the most satisfactory height appearing to be about one-half the depth of the tail water at maximum discharge. The addition of a second weir below, of equal or greater crest elevation, still further checks the tendency toward concentration.

10. When a sufficient body of water is maintained above the weirs, both in depth and length, to insure the formation of

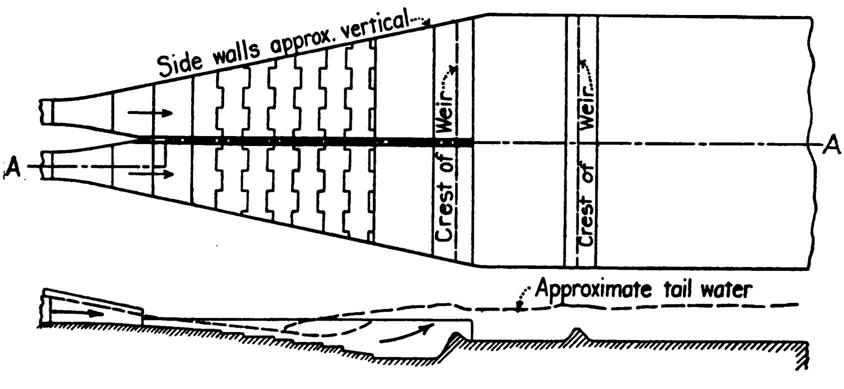


FIG. 37.—ADOPTED TYPE OF OUTLET STRUCTURE.

a jump, the depth of the channel below the weirs depends only upon the mean velocity which may be allowed on the material of the channel bed.

11. Roughening the floor of the channel above the jump, increases the channel friction, diminishes the velocity, leaves less work for the jump to do, and increases its stability.

12. The use of a baffle weir in the floor tends to cause the water to rise in the air in a sheet, eliminating the jump. This must be prevented by maintaining an adequate depth of tail water over the weir, to which end a second weir contributes, and by checking the velocity as much as possible above the jump by roughening the floor.

These conclusions are embodied in the form of outlet shown in figure 37, which was developed for use with twin conduits. It can be readily modified so as to provide for one, or for more than two conduits.

Other methods of dissipating energy may perhaps be employed to better advantage with smaller discharges or under special conditions. Sections XVIII to XX describe the results of experimental studies upon other types.

The investigations showed that the hydraulic jump, when properly controlled, is a most convenient and practicable method of eliminating the energy of flowing water. That its use has not been more extensive in the past has been largely due to uncertainty in regard to the theoretic law, for until recently there was divergence among authorities, and experimental data were inconclusive. The recent experiments of Professor Gibson, in conjunction with those herein described, are believed to have dispelled this uncertainty and to have opened up a promising field to the designer of hydraulic works.

SECTION XV.—EXPERIMENTAL PLANT

The experimental plant used in the summer of 1916 is shown in figures 38 and 39. During the fall of 1915, no baffle tank was used and a single conduit entered one end of the model tank. Figure 40 shows the interior of the model tank with the single conduit in the background and a channel in the center arranged for the study of the law of the hydraulic jump. Other arrangements of the interior were made from time to time.

Water was supplied by a 14-inch double suction centrifugal pump, driven by a 50 H. P. induction motor. The delivery main was 14 inches in diameter and included a Venturi meter with a Simplex Manometer, the latter being housed in the small building adjoining the pump house. Gate valves in the suction pipe and beyond the meter permitted regulation of the flow, which

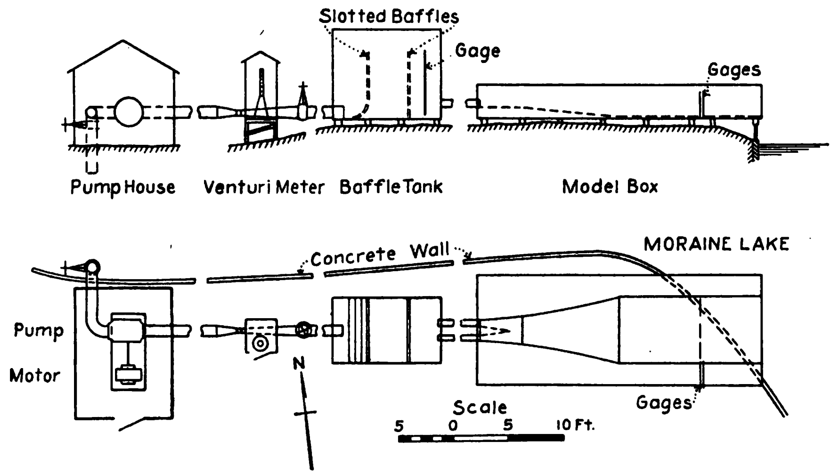


FIG. 38.—PLAN AND ELEVATION OF EXPERIMENTAL PLANT.

could be read by the scale of the manometer up to 10 second feet. For the single conduit experiments a short transition section of galvanized iron connected the 14-inch round pipe with the model tank, the outlet end being a reproduction on a one-sixteenth scale of the conduit tentatively designed for the Germantown dam, this being a three centered horseshoe shaped arch with a dished invert.

The model tank was rectangular in form, open at the end opposite the conduit entrance, and originally 21 feet long; the depth of water in the tank could be regulated by weir boards at the open end. In 1916 it was lengthened by 6 feet and the

FIG. 39.—GENERAL VIEW OF EXPERIMENTAL PLANT.

FIG. 40.—INTERIOR OF MODEL TANK.

baffle tank was constructed. Two short conduits with rounded entrances led from the baffle tank to the model tank. The experiments upon the control of the hydraulic jump were mainly conducted in this modified plant.

Discharges were measured by the Venturi meter; velocities, when permitted, by a small Price meter, and, when the depths were too small and the values too high for the use of the meter by Pitot tubes. The Venturi meter was calibrated by the use of a sharp edged weir in the model box, and found to give values correct within three per cent. The current meter had been rated at the Rennselaer Polytechnic Institute, and the Pitot tubes were calibrated on the work. In the fall of 1915, glass tubes were used, these being bent and drawn down to a point. They were calibrated by making velocity cross sections of flow, with the tube, and computing the discharge. These discharges when referred to the Venturi meter readings were uniformly too high, and in about the same ratio for different discharges and different sizes of tube. Eight observations on three tubes indicated that the velocity measurements obtained by the tubes were to be multiplied by the factor 0.875 to obtain true velocities.

Later a brass blow pipe tube was adapted by connecting its bent tip to a glass tube. This tube when calibrated in 1916 by the flow from a sharp edged orifice in the baffle tank gave a coefficient of unity. Unfortunately the glass tubes were fragile and easily destroyed, and none of them were available after the orifice was provided.

Depths were measured by a contact gage, shown in figure 40, consisting of a steel rod attached to a vertical scale with a vernier, all being supported upon a beam across the model tank. The bottom of the rod was rounded at first, but in 1916 was cut to a sharp point. This gage when used on water at high velocity tended to give results too large, inasmuch as the surface was always somewhat uneven in character and the tendency was always to touch the high places. In the 1916 experiments, depths were measured in the lower part of the model box by two piezometers connected to the floor of the channel at each side thereof. These observations were more satisfactory.

Insofar as observations were used to check theoretic formulas, the apparatus was too crude to secure results of high precision, as will be observed below in the discussion of experiments upon the hydraulic jump. There was also so much tur-

bulence in the water, in the greater number of the experiments; that most measurements were necessarily approximate. The current meter, for instance, was used occasionally where the turbulence was so great that the discharge computed from its indications were over 50 per cent above the known values. However, the experiments were primarily qualitative in character, and aimed to test the comparative behavior of various types. From this point of view, much latitude in the precision of measurements was allowable.

In the following discussion, a large amount of material has been condensed, and only essential points have been treated. The actual work of performing the experiments occupied nearly five months, while the office investigations and preparation of reports consumed several months more. All important conclusions were verified by many observations, and detailed measurements in experiments of importance were always checked. The relation of the work to its ultimate use in design was constantly kept in mind, and observations were always made of peculiarities which would be of importance in the design of the works, even though they might not be expressed in definite terms. It is believed that the work is valuable and reliable.

SECTION XVI.—THE HYDRAULIC JUMP

Although, as stated above, the experiments were initiated with a model of baffle piers, with other devices planned as alternatives, the hydraulic jump was soon seen to offer promises of successful solution and the investigation was finally concentrated upon this phenomenon. It is therefore deemed best to neglect the chronologic order of the work in this presentation, and to discuss the studies of the jump primarily, with additional notes upon the other types studied.

In the hydraulic jump water at comparatively high velocity and small depth passes to a stage of low velocity and great depth controlled by structures or other conditions in the channel below. It is characterized by a rise in the surface of the water and by more or less turbulence and loss of head, the degree of the latter depending upon the magnitude of the jump. The jump regularly occurs at the foot of spillway dams and is often seen when water discharges through a sluice gate into a channel of the same width as the gate.

The jump has been studied experimentally and theoretically for over a hundred years with rather inconsistent results. Several

theoretic treatments have been developed showing the relations between velocities and depths above and below the jump. Of these the soundest is based upon the law of conservation of momentum, and was first advanced by Belanger in 1838.* Among English writers it is generally ascribed to Professor Unwin,† although others have developed the same treatment independently. The law of the jump in rectangular channels is most frequently stated thus:

$$D_2 = -\frac{D_1}{2} + \sqrt{\frac{D_1^2}{4} + \frac{2D_1V_1^2}{g}},$$

where D_1 = depth above the jump,
D_2 = depth below jump,
V_1 = velocity above the jump,
g = acceleration of gravity.

The demonstration of this formula is given in the paper by Professor S. M. Woodward, entitled "Theory of the Hydraulic Jump and Backwater Curves," published herewith, where some analysis is also given of the behavior of the water within the jump.

The above equation, which is herein called the *momentum formula*, may be readily changed to the form

$$J^2 + J = 4\frac{H_1}{D_1},$$

where $J = \dfrac{D_2}{D_1}$,

$H_1 = \dfrac{V_1^2}{2g}$ = velocity head above the jump.

The physical significance of the law is more apparent in this form, for it is seen that the greater the ratio of velocity head to depth above the jump, the greater must be the jump. The curve of this equation is plotted on figure 41, so as to compare

*See Bresse, Cours de Méchanique Appliquée, Paris, 1860. Pt. II, Sec. IV.
†See Encyclopedia Britannica, 9th Edition, Vol. 12, p. 499.

readily the various experimental data available. With it is plotted the equation

$$J^2 - \frac{H_1}{D_1} J = \frac{H_1}{D_1},$$

which would govern the relations between velocity and depths if there were no loss of energy in the jump. Experiments which plot below this line indicate a loss of energy, while those plotting above it indicate a gain, which is impossible. The credibility of various experiments is largely determined by reference to this line. It will be observed that the two curves coincide when $J = 1$ and $H_1/D_1 = 0.5$, that is, at the critical depth.

Recorded Experiments Upon the Hydraulic Jump

The earliest recorded experiments are those of Bidone,* made in 1818 in a rectangular trough 1.066 feet wide. These have been frequently quoted, but the magnitude of the jump observed was too small to afford verification of any law. Moreover, his experiments, as well as others, suffered from the difficulty of measuring the physical quantities with accuracy. His results are plotted upon figure 41, where it is seen that they tend to fall outside the limiting curve of no loss of energy. His results are also given in table 1.

Table 1.—Bidone's Observations on the Hydraulic Jump

Series	Observations in Series	D_1 Feet	V_1 Feet per second	D_2 Feet	J	H_1 Feet	$\frac{H_1}{D_1}$
I	4	0.154	4.47	0.431	2.80	0.310	2.02
II	5	.209	5.57	.630	3.01	.481	2.30
III	3	.243	6.35	.749	3.08	.627	2.58
IV	4	.150	4.57	.414	2.76	.324	2.16

In 1856 Darcy and Bazin made some observations in a rectangular timber conduit 6.53 feet wide.† They were subject to the same limitation of small amplitude, and some of the observations fall outside of the limiting curve. They have not been included in figure 41, because it was felt that their inclusion would shed no light upon the verification of the law.

Some observations were made at Lehigh University, in 1894

*Transactions, Royal Society of Turin, 1819, pp. 21-80.

†See Recherches Hydrauliques, Paris, 1865, pp. 284-292.

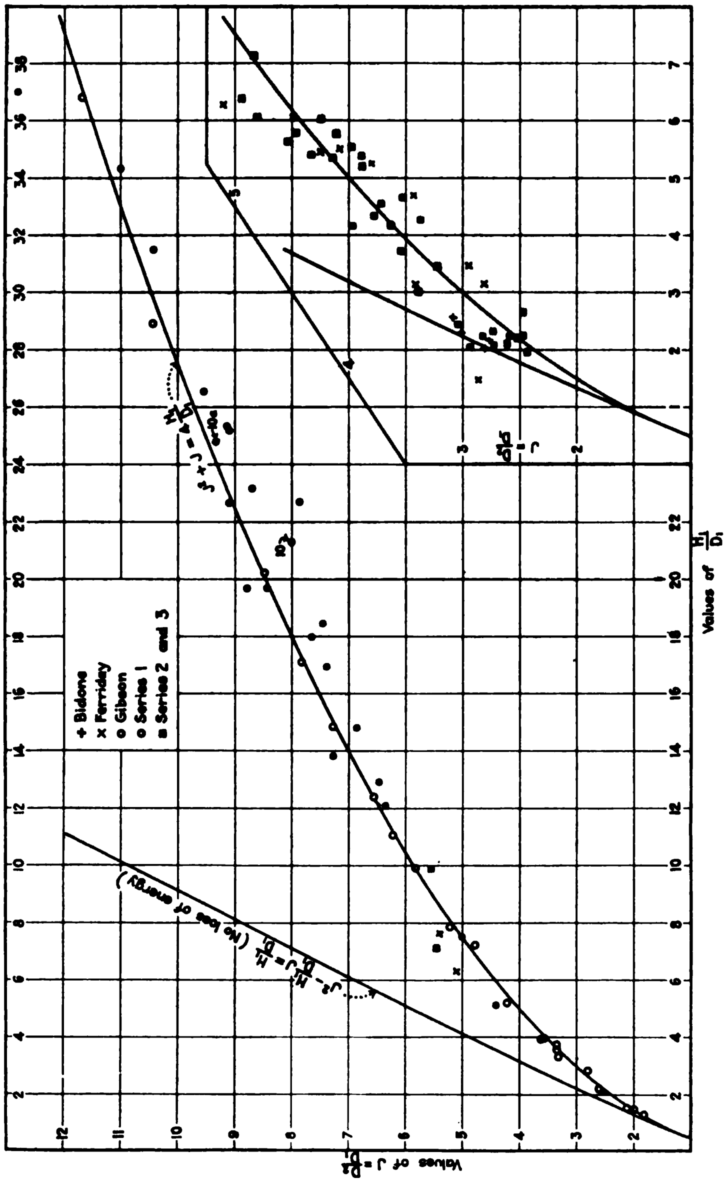


FIG. 41.—DIAGRAM SHOWING AGREEMENT BETWEEN MOMENTUM FORMULA AND EXPERIMENTAL DATA.

by Robert Ferriday, in a trough 0.66 foot wide, in which greater amplitudes were reached.† These are given in table 2 and are platted on figure 41, where they are seen to verify the momentum formula substantially.

Table 2.—Ferriday's Observations on the Hydraulic Jump

Series	Observations in Series	D_1 Feet	V_1 Feet per second	D_2 Feet	J	H_1 Feet	$\frac{H_1}{D_1}$
I	3	0.050	2.18	0.143	2.86	0.074	1.48
II	5	.044	2.98	.150	3.41	.138	3.14
III	6	.036	3.56	.153	4.25	.196	5.45
IV	5	.033	3.66	.168	5.10	.207	6.28
V	3	.095	4.39	.267	2.81	.298	3.14
VI	4	.083	5.02	.285	3.44	.390	4.70
VII	3	.071	5.02	.290	4.08	.390	5.50
IX	4	.055	3.50	.162	2.95	.190	3.46
X	4	.046	3.95	.175	3.80	.242	5.26
XI	5	.042	4.06	.188	4.48	.255	6.08
XII	4	.038	4.33	.205	5.40	.290	7.64

However, the best verification of the momentum law is afforded by the experiments of Professor A. H. Gibson,‡ of Dundee, Scotland. His work was carried on in a flume 3 feet wide, the height of tail water being determined which would bring the jump close to a rectangular opening at the bottom of the flume, corresponding to a sluice gate. His observations have a greater degree of precision than any previous ones, and also extend through a greater range. The results are given in table 3 and the verification of the law which they furnish is almost startling, see figure 41.

The investigations of the Miami Conservancy District had developed a number of observations upon the jump when the discussion of Kenninson's paper in the Proceedings of the American Society of Civil Engineers called attention to the previously unknown work of Gibson and Ferriday. The work of the District, in connection with that of the other investigators, is held to establish completely the accuracy of the momentum formula, and being hitherto unpublished is described below in detail. As will appear, the methods employed were not of the order of precision obtained by Gibson, but the fact that they

†See Transactions, Am. Soc. C. E., Vol. 80, p. 385.

‡Proceedings Inst. C. E., Vol. 197, p. 233. Also Trans. Am. Soc. C. E., Vol. 80, p. 413.

were obtained in a channel of different form and with greater depths in many cases, increases their value.

Table 3.—Gibson's Observations on the Hydraulic Jump

Series	D_1 Feet	V_1 Feet per sec.	D_2 Feet	J	H_1 Feet	$\frac{H_1}{D_1}$
A	0.0735	4.30	0.265	3.61	0.288	3.92
	.0731	5.82	.350	4.79	0.526	7.20
	.0730	7.20	.455	6.23	0.805	11.03
	.0729	8.33	.530	7.28	1.079	14.82
	.0728	8.95	.570	7.83	1.245	17.11
	.0730	9.74	.612	8.39	1.474	20.22
	.0728	11.66	.760	10.44	2.110	29.00
	.0727	13.12	.850	11.69	2.676	36.80
B	0.1465	3.45	0.267	1.82	0.185	1.26
	.1395	5.68	.467	3.35	0.501	3.59
	.1390	6.82	.587	4.22	0.723	5.20
	.1390	8.39	.726	5.22	1.093	7.86
	.1390	9.40	.808	5.81	1.372	9.88
	.1390	10.53	.910	6.55	1.725	12.41
C	0.2075	4.43	0.419	2.02	0.305	1.47
	.2070	4.53	.440	2.12	.318	1.54
	.2048	5.34	.530	2.59	.443	2.16
	.2046	6.09	.575	2.81	.575	2.82
	.2040	6.63	.678	3.32	.684	3.36
	.2043	7.05	.684	3.34	.772	3.77
	.2043	7.24	.735	3.59	.813	3.98

Researches of The Miami Conservancy District

The experiments upon the height of jump were arranged in three series, which differ as to the form of channel and the methods of measurement, see figure 42. In Series 1 the discharge of the conduit was permitted to spread over the floor of the model tank in a thin sheet, a jump being formed when the water level in the downstream end of the tank was raised by the insertion of weir boards. Figure 43 shows the nature of the flow without a weir, with surface contours and cross sections, and indicates that the sheet becomes nearly uniform in depth as the distance from the outlet increases. The position of the jump was observed to shift materially with small changes in the height of the outlet weir and in the consequent depth of water. When the depth was made so great as to cause the jump to move into the spreading stream, the jump lost its character, and turmoil and concentration of flow ensued in the lower part of the tank.

In Series 1 the observations on the height of the jump were made on the center line of the model tank, being confined to cases where the jump was straight and at right angles to the center line of the tank. Depths were determined with the contact gage, and the velocity above the jump was measured with glass Pitot tubes.

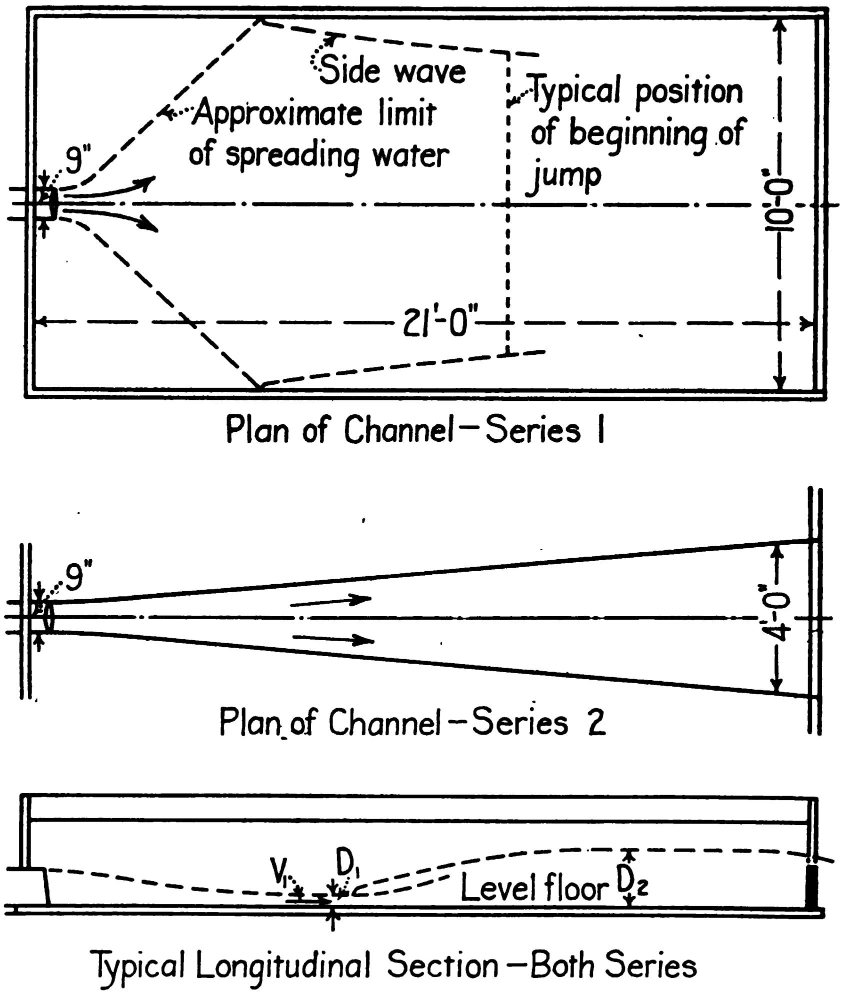


FIG. 42.—FORMS OF CHANNEL USED IN EXPERIMENTAL STUDY OF THE HYDRAULIC JUMP.

Measurements were taken at vertical intervals of not more than 0.02 foot and to the mean velocity was applied the coefficient of 0.875 to obtain the true mean velocity V_1, see page 11. The observations of this series are summarized in table 4 and are plotted on figure 41, the former giving the position of the beginning of the jump, with reference to the inlet end of the

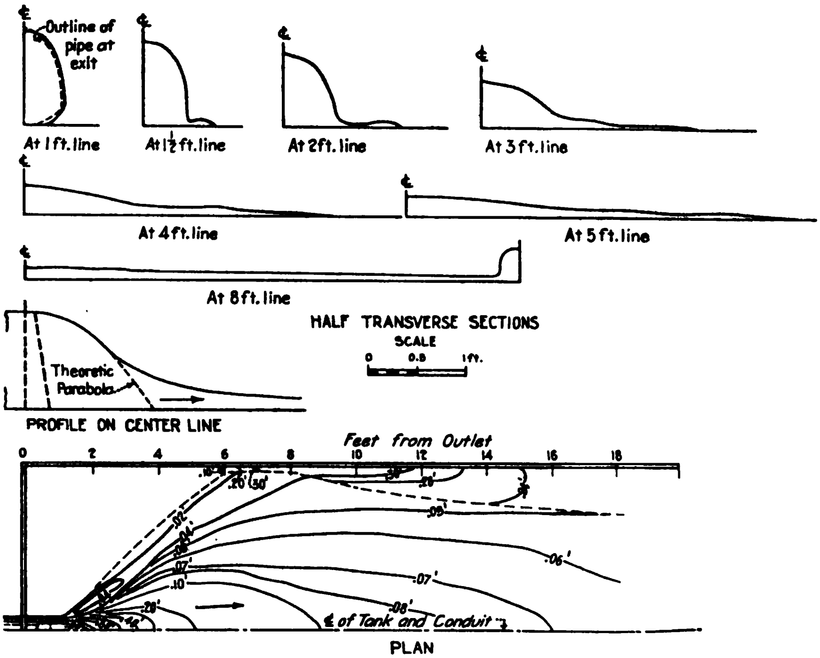


FIG. 43.—CONTOURS AND CROSS SECTIONS OF OUTFLOWING WATER, SERIES 1.

Jet emerges on smooth level floor and gradually spreads laterally until it strikes side walls, where it produces a marked roll.

model box, as well as the observed and computed quantities. It is to be noted that the values of H_1/D_1 range from 5.12 to 37.00 in this table, while the value of $J\,(=D_2/D_1)$ are from 2.54 to 12.80. These apply to a far wider range of channel conditions than do the results of any other experiments except Gibson's.

The experiments of Series 1 give points which verify the theoretic curve fairly well, although most of them fall below it. This discrepancy is believed to be largely explained by the depth measurements, which are believed to have been generally too high

Miami Conservancy District's Observations on the Hydraulic Jump, Series 1

D_1 Feet	V Feet per second	D_2 Feet	J	H_1 Feet	$\frac{H_1}{D_1}$
0.050	4.07	0.22	4.40	0.256	5.12
.050	4.94	.25	5.00	0.377	7.54
.055	6.85	.40	7.27	0.725	13.18
.065	8.44	.48	7.39	1.100	16.93
.055	6.56	.35	6.36	0.666	12.12
.070	8.16	.48	6.86	1.037	14.81
.050	7.97	.44	8.80	0.984	19.68
.065	7.36	.42	6.47	0.839	12.92
.220	5.54	.56	2.54	0.479	2.16
.070	9.79	.56	8.00	1.490	21.30
.045	10.36	.58	12.80	1.665	37.00
.085	11.15	.67	7.88	1.930	22.70
.075	11.07	.68	9.13	1.900	25.35
.060	11.53	.66	11.00	2.060	34.33
.085	9.90	.65	7.65	1.525	17.95
.090	10.68	.76	8.44	1.770	19.70
.085	11.26	.74	8.70	1.968	23.18
.110	11.43	.82	7.46	2.030	18.45
.090	11.46	.82	9.10	2.040	22.67
.100	12.72	.91	9.10	2.520	25.20
.110	13.70	1.05	9.55	2.920	26.55
.110	14.93	1.15	10.44	3.465	31.50

...above the jump, as explained on page 70. Sufficient data was not available to determine a definite depth correction. However, if any observed value of D_1 be supposed to be diminished by a definite amount it is obvious that the corresponding value of J will be increased more than the corresponding value of H_1/D_1, and the plotted point will fall closer to the curve. For instance, in experiment 10, where $V_1 = 9.79$, and $D_2 = 0.56$, let D_1 be assumed to have the value of 0.06 instead of 0.07. Then $J = 9.33$ and $H_1/D_1 = 24.8$. The point representing these values plots close to the curve, see 10a on figure 41. It has not been considered justifiable to apply an arbitrary correction to all the observations, but the existence of the above tendency explains the discrepancy.

The experiments of Series 2 were made in a trough whose vertical sides were tangent to the conduit and flared slowly in the manner shown in figure 42. The floor of the trough was ... and the depth of water was regulated, as before, with ... In this series the depths were measured above ... jump with the contact gage, the mean of several

measurements across a section being taken as the mean for that section. The mean velocity above the jump was obtained by dividing the discharge by the area of the cross section. Several observations were taken with the jump close to the conduit outlet, but were discarded because the depth measurement was uncertain and because it was believed that the water had not adjusted itself to the changed channel section.

These experiments seem more consistent than do those of Series 1, the principal reason probably being that the depth determinations above the jump were more accurate. The mean velocity above the jump is also more accurate, while the depths below are no less so. The latter measurement is not so simple as might appear, for waves and pulsations continue sometimes for a considerable distance below the jump, and the lower end

Table 5.—The Miami Conservancy District's Observations on the Hydraulic Jump, Series 2

Run	Position of Jump. Feet	D_1 Feet	V_1 Feet per second	D_2 Feet	J	H_1 Feet	$\frac{H_1}{D_1}$
100	5.5	0.32	6.60	0.84	2.62	0.675	2.11
101	11.0	.22	5.73	0.60	2.73	0.510	2.32
102	9.0	.24	6.15	0.73	3.04	0.586	2.44
111	14.2	.17	7.95	0.70	4.11	0.980	5.76
112	10.0	.24	7.57	0.85	3.54	0.890	3.71
113	7.5	.34	6.68	1.00	2.94	0.691	2.03
114	5.4	.39	7.27	1.06	2.72	0.818	2.10
119	4.3	.48	8.35	1.35	2.82	1.080	2.25
120	9.3	.31	7.75	1.05	3.39	0.932	3.01
121	13.3	.21	8.52	0.87	4.14	1.125	5.36
122	12.5	.24	9.45	1.07	4.46	1.385	5.78
124	9.0	.32	9.25	1.27	3.97	1.33	4.15
125	6.3	.38	10.10	1.38	3.63	1.580	4.16
129	3.7	.55	11.05	1.77	3.22	1.890	3.44
130	6.2	.42	10.83	1.59	3.78	1.820	4.34
131	8.8	.33	10.72	1.43	4.33	1.780	5.40
132	11.7	.27	10.28	1.30	4.81	1.640	6.07
135	11.8	.29	10.93	1.43	4.94	1.850	6.38
136	9.0	.35	11.26	1.59	4.54	1.965	5.62
137	6.5	.40	12.50	1.70	4.25	2.420	6.04
138	4.5	.49	12.83	1.90	3.88	2.550	5.20
139	2.0	.85	10.37	2.07	2.44	1.66	1.95
142	2.0	.87	11.18	2.24	2.58	1.93	2.23
143	4.5	.52	13.62	2.07	3.98	2.875	5.54
144	8.5	.36	12.88	1.74	4.83	2.560	7.12
145	11.0	.34	11.12	1.54	4.53	1.920	5.65
146	13.5	.30	11.78	1.64	5.47	2.150	7.17
147	9.5	.33	14.50	1.83	5.55	3.26	9.83
148	5.0	.54	13.70	2.10	3.89	2.910	5.39
149	3.5	.64	13.82	2.25	3.52	2.960	4.63
150	3.0	.69	13.77	2.32	3.36	2.94	4.26

of the jump is not capable of definite determination. Table 5 gives these results. It should be noted that the depths D_1 are much greater than in Series 1, ranging from 0.17 to 0.64, more closely resembling the conditions of practical use. Yet the theoretic law is better verified by these observations than by those of Series 1.

The experiments listed as Series 3, given in table 6, are a few miscellaneous ones. Those marked b-3 and b-4 were similar

Table 6.—The Miami Conservancy District's Observations on the Hydraulic Jump, Series 3

Run	D_1 Feet	V_1 Feet per second	D_2 Feet	J	H_1 Feet	$\frac{H_1}{D_1}$
63	0.53	8.71	1.33	2.51	1.175	2.22
64	0.58	13.00	2.16	3.72	2.63	4.54
7a	0.195	5.78	0.48	2.46	0.517	2.65
8a	0.19	5.25	0.47	2.48	0.427	2.24

to those of Series 2, except that the sides of the trough were parallel. The measurements were made in the same way. Those marked 7-a and 8-a were made in connection with the experiments 7 and 8 of Series 1, but were taken at the side of the model tank where a side roll was established, as shown in figure 43. The observations are plotted on figure 41 where their substantial agreement with the momentum formula is observed.

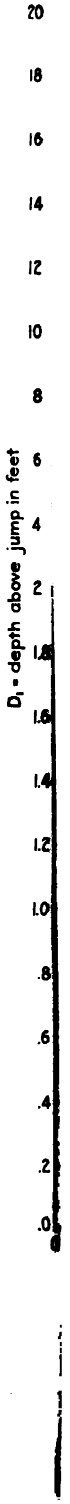
To more readily apply the jump formula figure 44 has been devised. This gives curves showing relationships between V_1, D_1, V_2, and D_2, such that by entering the diagram with any two of these quantities the other two may be obtained at once. The use of this diagram avoids all calculations within the limits of its range. Its construction is easily understood. For example, let D_2 be assumed as equal to 1 in the equation

$$D_2 = -\frac{D_1}{2} + \sqrt{\frac{D_1^2}{4} + \frac{2V_1^2 D_1}{g}}.$$

Then the equation becomes

$$\frac{1}{D_1} = \frac{2V_1^2}{g} - 1,$$

which is plotted on figure 44, as the line $D_2 = 1$. A similar procedure is followed for the other curves.


20
18
16
14
12
10
8
6
4
2
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
D_1 = depth above jump in feet

It cannot be denied that the experiments cited in support of the formula are upon a small scale, and that after all they cover a comparatively limited range of velocities and depths. This range is shown graphically upon figure 45, where all the results of Gibson, Ferriday, and the District are plotted. Inspection of this diagram for verification of the law is not so easy as in figure 41, but it is inserted to show the limits of the experi-

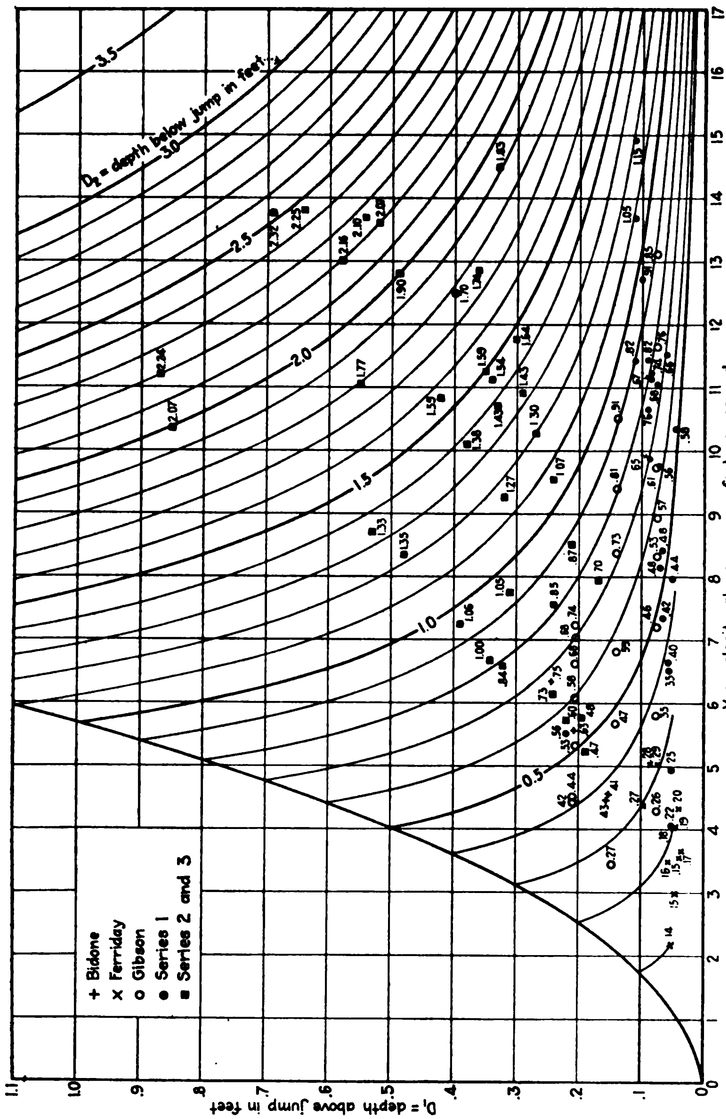


FIG. 45.—DIAGRAM, SIMILAR TO PART OF FIGURE 44, SHOWING RANGE OF PUBLISHED EXPERIMENTS UPON THE HYDRAULIC JUMP.

Numbers at plotted points are observed values of D_2. Curves show theoretic values of D_2.

ments. The applicability of the formula to conditions outside those of the experiments rests largely upon the fundamental law of conservation of momentum. The law of no loss of energy manifestly does not apply and all others advanced, except the momentum formula, are more or less empirical.

Other conclusions deduced from the experiments are as follows:—

(a) The hydraulic jump can be used as a safe method of reducing the velocities below the conduit outlets to such values that the current will not dangerously erode the natural channels.

(b) On a level floor with low friction, for a slight change

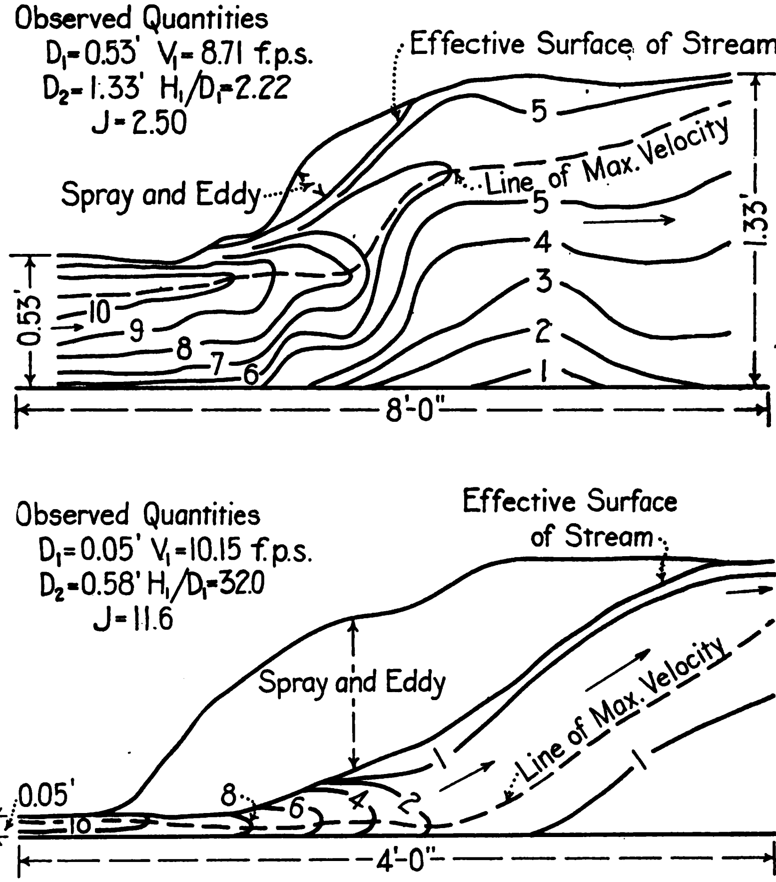


FIG. 46.—LONGITUDINAL SECTIONS OF HYDRAULIC JUMPS SHOWING LINES OF EQUAL VELOCITY.

Numbers on lines are velocities in feet per second

in the depth or velocity, the position of the jump will vary through a wide range. Inasmuch as the jump must occur in the regulating works under a wide variation of discharges and depths, this was objectionable.

(c) The length of the jump is approximately five times its height. While the beginning of the jump is fairly definite, its lower end is indefinite, and this figure represents merely a general estimate. The lower end was taken as the place where the water surface became and remained sensibly level, a place which was variable in position and difficult to locate.

(d) For low values of H_1/D_1 the loss of head in the jump is small, while for high values the loss is great. This loss is due to the disturbances in the jump; and, for any value of H_1/D_1, it is indicated, in a way, by the difference in vertical ordinates between the curves of figure 41.

(e) The velocity of the water gradually diminishes through the jump, as is indicated by the profiles in figure 46. These show lines of equal velocity for jumps of quite different amplitude. The profiles are typical of a number of similar ones, all of which show the region of spray and foam above an expanding stream, the surface of which could always be felt with the contact gage. Velocities were measured with the Pitot tube, and are only roughly approximate on account of the influence of cross currents and eddies in the water.

Before passing on, the profile on the center line in figure 43 should be noted. The surface coincides for some distance from the outlet with the theoretic parabolic path of water discharging from the outlet as from an orifice. The same agreement was noted in later experiments with a discharge of rectangular section. This observation is of use in calculating the curve which defines the limits of the spreading water.

SECTION XVII.—CONTROL OF THE HYDRAULIC JUMP

The established theory of the jump gave the depth of water below, herein called the tail water, which will cause a flow of given initial depth and velocity to form a jump. The next problem was to adapt the jump to the conditions of outlet discharge, that is, to determine such a form of outlet channel as would insure and localize the formation of the jump. The position of the jump could not be closely predicted when the floor was level and smooth.

Floor Roughened with Small Blocks

Among the devices considered for the reduction of velocity was that of studding the bottom of the channel with knobs, boulders, or small piers. To study the probable effects of these upon the jump, experiments were made with the floor studded with small ¾-inch blocks as shown in figure 47, the side of the channel being shaped so as to conform closely to the natural spread of a discharge of 8 second feet; see also figure 43. It was found that the roughened floor, while eliminating a material portion of the kinetic energy of the water caused the jump to be less variable in position under different discharge and tail water conditions. It also tended to cause a more uniform depth above the jump. Figure 47 shows typical observations on the distribution of velocity below the jump, for flow of this character.

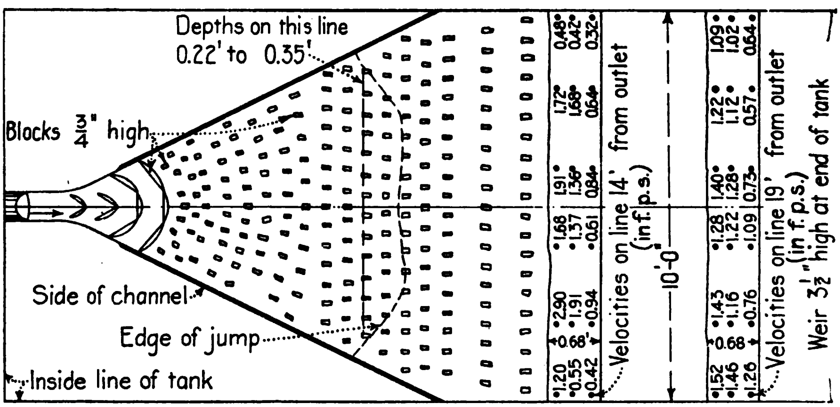


FIG. 47.—PLAN OF MODEL CHANNEL SHOWING VELOCITIES BELOW JUMP.

Discharge 8 second feet; floor level and studded with ¾-inch blocks.

In order to study still further the effect of roughening the floor, the side walls were modified by sloping them at 3 on 1, and drawing them in, as shown on figure 48, this arrangement conforming more closely with the probable plan of the outlet channel. With this form three series of experiments were conducted, namely Series 4, with a smooth floor; Series 5, with the entire floor studded with ¾-inch blocks; and Series 6, with 1½-inch blocks. In each of these series, runs were made with discharges of 7, 8, 9, and 10 second feet and weir heights

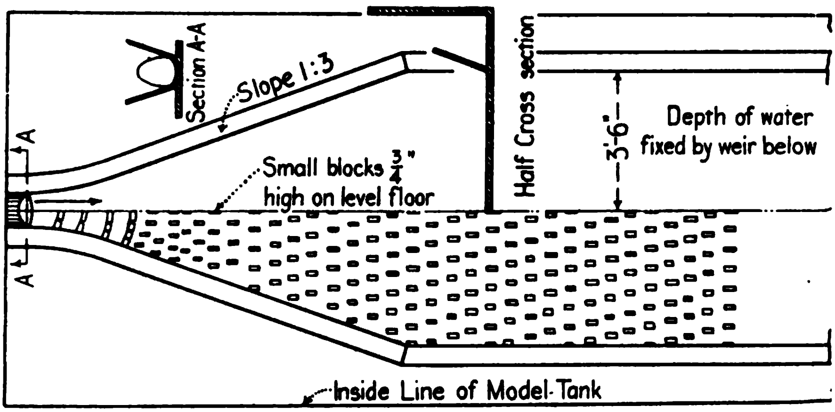


FIG. 48.—PLAN OF FIRST MODEL OF OUTLET STRUCTURE FOR GERMANTOWN DAM.

at the end of the tank of 2¾, 4¼, 6¼, and 8¼ inches, each series therefore including 16 runs.

Figures 49 and 50 show the performance at 10 second feet with the greatest weir height in Series 4. When the discharge

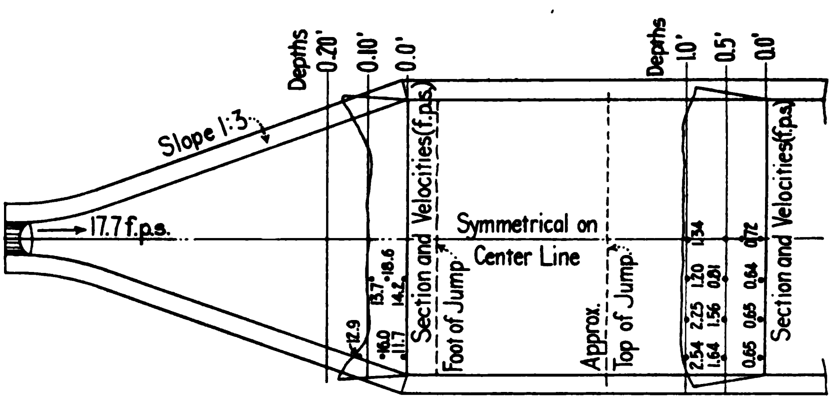


FIG. 49.—PLAN OF FIRST MODEL OF GERMANTOWN DAM OUTLET SHOWING VELOCITIES BELOW THE JUMP.

Discharge 10 second feet; smooth floor.

was diminished the jump traveled upstream and finally was drowned out more or less completely, with the production of an eddy below, as shown in figure 51. The same change took place at lower weir heights, but when the weirs became quite low and the discharge was high, the water rose over the end weir without a jump.

In Series 5, with the ¾-inch blocks in position and the maximum discharge and tail water, the same drowning out and eddying occurred, see figure 52. A straight jump was obtained only when the weir height was reduced to 4¼ inches, see figures 53 and 54, when the behavior of the jump was substantially as in Series 4. The obvious explanation is that the velocity V_1 is so reduced by friction that a smaller depth below is required to develop the jump. An excess of tail water on the horizontal floor therefore will tend to produce concentration of flow without a true jump..

In Series 6, with 1½-inch blocks, the same performance was observed with high discharge and tail water. As the weir height was lowered the water tended to shoot down the center of the channel at high velocity, the jump being notched far downstream in the center and ill defined.

The following conclusions were drawn:

1. With the blocks in use the bottom velocities below the jump are diminished but the surface velocities tend to be higher.

2. The jump occurs sooner with the blocks than without them under the same tail water conditions.

3. When the tail water is increased, with the blocks in use, the main stream tends to go down one side of the channel in much the same way as in the smooth channel.

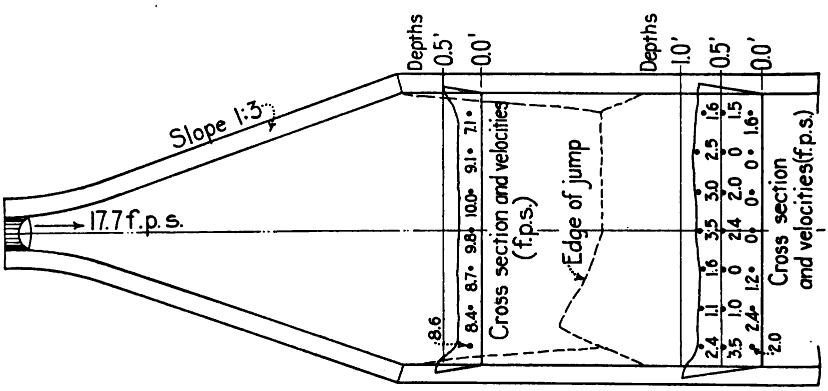


FIG. 54.—PLAN OF FIRST MODEL OF GERMANTOWN DAM OUTLET SHOWING VELOCITIES BELOW JUMP.

Floor studded with ¾-inch blocks. Discharge 10 second feet.

It was believed that placing obstructions of this character on the bottom would lead to severe impact stresses, and that the effects of the obstacles were so complicated and obscure when

FIG. 50.—VIEW OF TYPICAL JUMP IN SERIES 4, DISCHARGE 10 SECOND FEET.

First model of Germantown Dam outlet; smooth floor and 8¼-inch regulating weir. Jump is uniform, symmetrical, and square with the sides of the channel, while the water flows away placidly below the jump.

FIG. 51.—VIEW OF JUMP IN SERIES 4, DISCHARGE 7 SECOND FEET.

First model of Germantown Dam outlet; smooth floor and 8¼-inch regulating weir. Due to the high tail water a modified jump occurred close to the end of the conduit, with an eddy blow it.

FIG. 52.—VIEW OF JUMP IN SERIES 5, 8¼-INCH WEIR.

First model of Germantown Dam outlet; floor studded with ¾-inch blocks; discharge 10 second feet. Jump becomes unsymmetrical and is finally drowned out when tail water surface is too high.

FIG. 53.—VIEW OF JUMP IN SERIES 5, 4¼-INCH WEIR.

First model of Germantown Dam outlet; floor studded with ¾-inch blocks; discharge 10 second feet. This was the maximum height of weir which would permit the formation of a jump.

considered in detail, that it was not possible to formulate any definite law concerning them. Moreover, their effects under high tail water conditions were not definite, and it seemed probable that in the large scale construction the bulk of the flow might pass over them without much reduction of energy. This device was therefore considered unreliable.

Channel with Sloping Floor

At this point experimental work was discontinued for the winter and the problem was taken up in the designing office. Tentative designs were developed for the outlets at the several dams. In each of these more than one conduit discharged into the channel, the number varying with the discharge and construction requirements at the several retarding basins. The walls of each channel were spread gradually from the outlets of the conduits, so as to permit the water to develop into a thin sheet, and a sloping floor was provided. Calculations based upon the theoretic law of the jump showed that the latter would form upon this slope under all possible conditions of discharge and tail water.

But certain questions as to the operation of the channel seemed to require answers, for example:

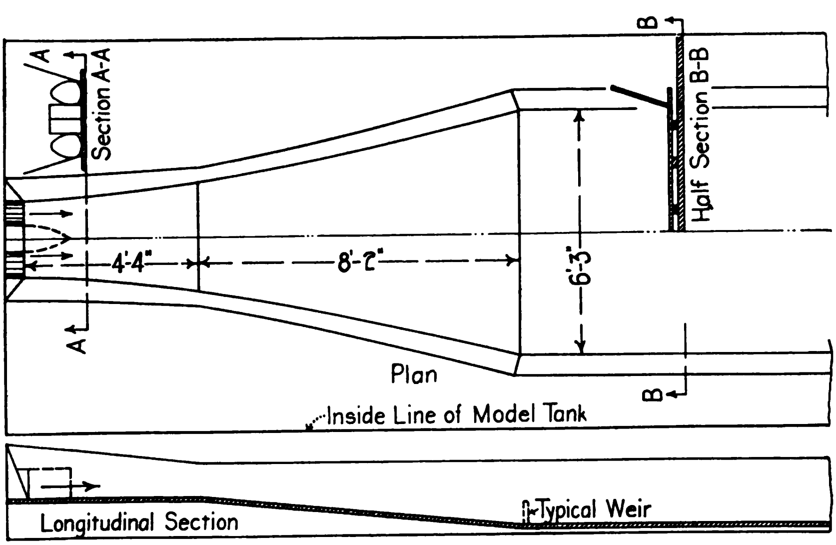


FIG. 55.—PLAN AND SECTIONS OF SECOND MODEL OF GERMANTOWN DAM OUTLET, SERIES A, SMOOTH FLOOR.

1. Would there be a reasonable agreement between the actual and computed position of the jump upon the channel bed?

2. Would the jump be stable, that is, would it be uniform across the channel, and would the tail water discharge be free from concentrations of velocity?

3. What would be the effect of the use of more than one conduit?

4. What would be the effect of partly or completely blocking one conduit?

5. What would be the effect of obstructions in the channel?

The conclusions reached from the studies made for the purpose of answering these questions are embodied in Section XIV. They are important because they represent the application of the jump theory to the problem of design and because they are believed to be demonstrated with reasonable conclusiveness.

The form of channel shown in figure 55 was used as a starting point for this investigation. It was an exact reproduction,

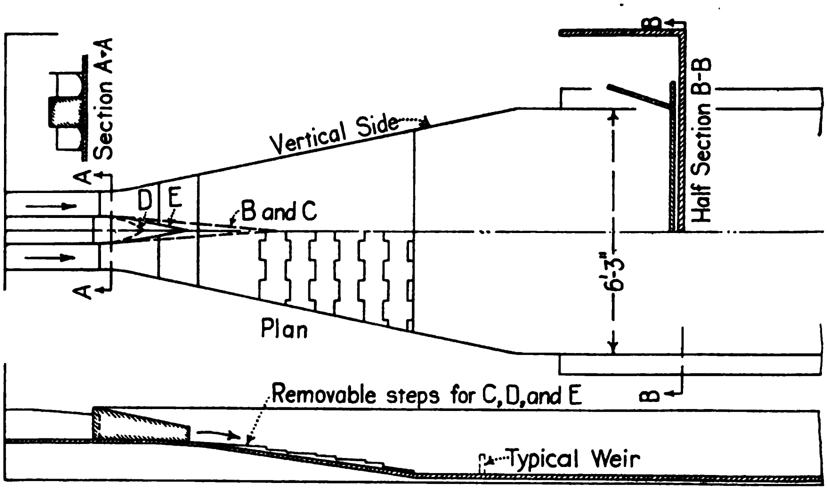


FIG. 56.—PLAN AND SECTIONS OF SECOND MODEL OF GERMANTOWN DAM OUTLET, SERIES B TO E, FLOOR STEEPER AND ROUGHENED.

on a scale of 1 to 16, of the tentative design for the Germantown outlet, which will have a capacity of 10,000 second feet when the basin is filled to the spillway level. The reduction in scale of 1 to 6 in linear dimensions invoved a reduction of 1 to 256 in the area of the channels, of 1 to 4 in velocity, and 1 to 16 in

velocity head; and accordingly the discharge corresponding to 10,000 second feet became 9.74 second feet, and the velocity of discharge 13.7 feet per second. These quantities were used in most of the experiments although others were employed at various times to check the behavior of the model under varying conditions of flow. In these experiments the baffle tank of figure 38 was employed to secure similar flow in the two conduits.

These experiments were arranged in six groups, in each of which the form of the sides and bottom of the channel were constant, while the discharge and tail water conditions were varied and obstructions of diverse character were employed in the channel. These series were characterized as follows:

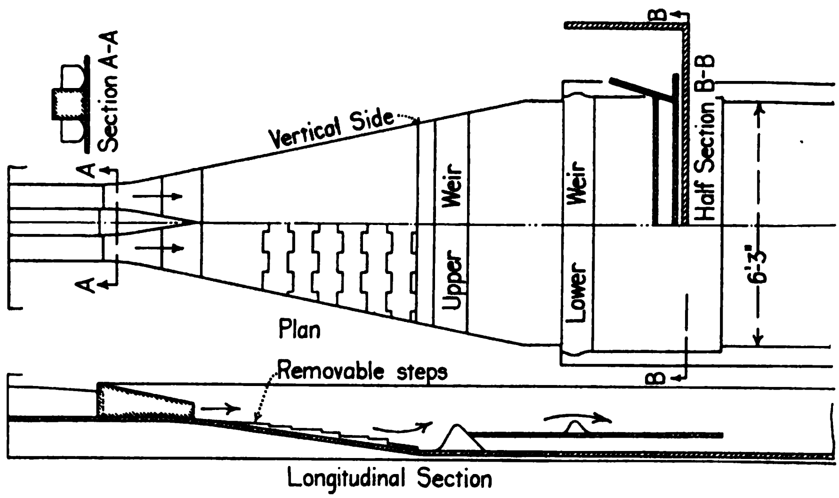


FIG. 57.—PLAN AND SECTIONS OF SECOND MODEL OF GERMANTOWN DAM OUTLET, SERIES F.

Ogee weirs and elevated floor installed below the jump.

(A) The channel was of the tentative form shown in figure 55.

(B) The slope of the bottom was steepened, the side walls made vertical and tangent to the conduit outlets whose shape was changed accordingly. A long, central nose was employed, as shown in figure 56.

(C) The sloping floor was roughened by a series of steps.

These were offset, as shown in figure 56, to prevent rhythmic vibration in the bottom. The channel otherwise was as in Series B.

(D) The long nose of Series B and C was replaced by a short stubby one.

(E) The stubby nose was replaced by one of medium length, and weirs of ogee section and varying height were placed in the channel below the slope.

(F) This was devoted to the effect of raising the channel floor below the 8-inch ogee weir, which was proved to be needed in the previous experiments, see figure 57.

Results of Various Series of Experiments

Series A. The observed position of the jump as compared with the computed position was of primary interest. In computing, an approximate surface curve was determined using an assumed value for the coefficient of roughness of the channel. The jump

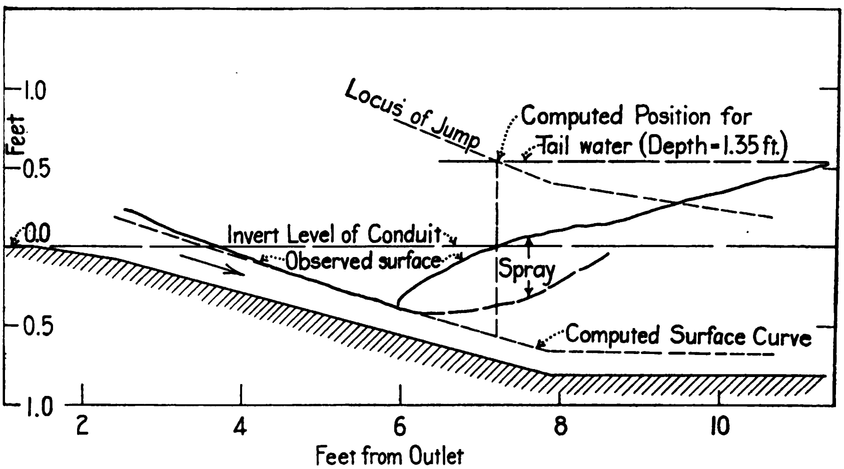


FIG. 58.—DIAGRAM SHOWING OBSERVED AND COMPUTED POSITIONS OF THE HYDRAULIC JUMP.
Series B; normal discharge of 9.74 second feet.

formula was then applied to the calculation of a line called the *locus of jump*, giving the depth of tail water necessary to produce a jump at any place with known depth and velocity. The intersection of the latter line with that representing the tail water level in any given case gives the position of jump, illus-

trated in figure 58. This diagram is made from observations in Series B, but fully illustrates the above remarks. In every case the jump began above the position computed and ended below it, as verified by many observations throughout the experiments. The end of the jump is indefinite, but it is clearly below the computed position. A reasonable assumption as to the place where the law of the jump applies is that it is the section where the velocity head of the expanding water has an average value. Inasmuch as the accurate measurement of velocity within the jump seems to be impossible, this cannot be verified.

The tentative design was not regarded as satisfactory in the region above the jump. The offset between the conduit outlet and the sides of the channel resulted in a side wave or roll, as illustrated in figure 59, which tended to rise above the channel walls. The short, stubby nose, illustrated in figure 55, had about the same effect as no nose at all, and was omitted in nearly all the experiments. The spreading water from the two conduits formed, at the line of junction, a rising jet, clearly seen in figure 59. The side rolls and jet resulted in an uneven sheet above the jump, with consequent irregularities below.

The latter were noticeable only when the discharge exceeded 6 second feet. Below this discharge the jump was straight and approximately at right angles to the center line of the channel, and the tail water was even and symmetrical in character. Above 6 second feet a concentration of flow in the center of the channel occurred and the jump was notched downstream in the center. A large eddy formed with velocities as high as 5 feet per second at the side of the channel when the normal discharge of 9.74 second feet was employed, as shown in figure 60. Inasmuch as the mean velocity of flow with the depth corresponding to this discharge should be about 1.10 feet per second, this condition was unsatisfactory.

Several devices were tried to check the concentration of flow, notably a partion wall in the center. Figure 61 shows the effect of placing central partitions of various lengths in the outlet. However, a far superior result was achieved by placing a 6-inch vertical weir in the channel at the foot of the slope.

An extensive series of observations was then conducted to determine the height of weir which would give the best velocity distribution below. In these observations the depth of tail water was varied with the discharge in proportion to the corresponding

variation in the full-sized structure at the Germantown Dam, thus:

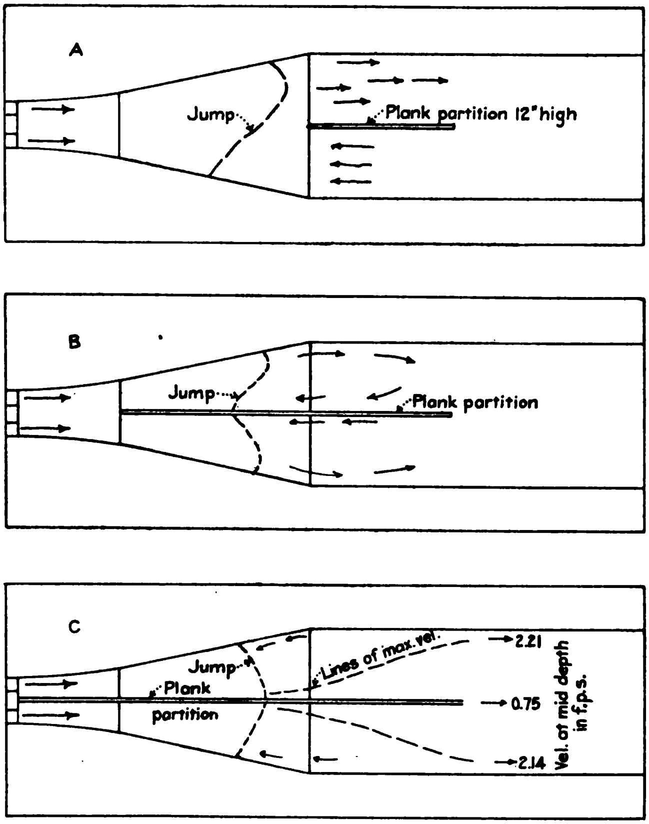


FIG. 61.—DIAGRAM SHOWING RESULTS OBTAINED BY INSTALLING A CENTRAL LONGITUDINAL PARTITION OF VARIOUS LENGTHS IN THE EXPERIMENTAL CHANNEL OF SERIES A.

Discharge, Second Feet	Conduit Velocity, Feet per second	Depth of Tail Water, Feet
4	5.6	1.05
6	8.4	1.14
8	11.3	1.25
9.74	13.7	1.33

The chief results of these experiments are shown in table 7, which gives values of the tail water velocity at a section 20.5 feet below the inlet end of the tank with various weirs. The

FIG. 59.—VIEW OF JUMP IN SERIES A, 4-INCH AND 6-INCH WEIRS.

Second model of Germantown Dam outlet, normal discharge of 9.74 second feet, and two submerged weirs 3 feet apart installed just below the jump. Upper weir 4 inches high, lower weir 6 inches high. These produced a symmetrical jump with uniform velocity distribution below them, but the offsets in the channel walls produced rolls of considerable height up the side walls, and the short nose caused the formation of a jet at the central line of impact.

FIG. 60.—VIEW OF JUMP IN SERIES A, NO WEIRS.

Second model of Germantown Dam outlet, normal discharge of 9.74 second feet. Jump was unsymmetrical, leading to considerable concentration of velocity, at one side of the channel below the jump.

FIG. 62.—VIEW OF EXPERIMENTAL CHANNEL WITH STEPPED SLOPING FLOOR, SERIES C.

The same model with smooth floor was used in Series B.

FIG. 63.—VIEW OF JUMP IN SERIES B, NO WEIRS.

Normal discharge of 9.74 second feet. With this discharge, or more, the jump began to be unstable and unsatisfactory.

mean velocities given are in general the mean of 7 determinations at equal intervals across the measurement section. Confirmation is offered by about 30 other experiments with various discharges, here omitted for the sake of brevity. It is seen that any submerged weir, 6 inches or more in height at the foot of the slope, equalized the flow sufficiently to prevent excessive velocities in the channel below. The best distribution was secured by placing a small weir on the slope preferably with a higher one below it. The latter method, however, was found to interfere with the jump and to produce a high boil over the weir. Upon the large scale, therefore, the jump might not be formed. Moreover, this weir was subject to greater velocities and higher impact effects than one farther downstream. It was believed preferable, therefore, to employ a higher weir located at or below the foot of the slope. These conclusions were confirmed by the observations at lower discharge, except that below 6 second feet, it seemed to make no difference whether a weir was employed.

The boiling condition mentioned above is generally present over the weir whenever one is used, although it is less in intensity as the height of the weir is diminished or as it is moved downstream. Lowering the tail water brings the jump downstream and finally causes it to be lost, the water then rising in a thin sheet from the weir in a manner similar to that described in Section XIX. It is essential to have high tail water over the weirs, and the latter may be regarded as regulating and stabilizing the tail water rather than as directly affecting the jump.

With one conduit blocked and partial discharge in the other, the water spread over the sloping floor and formed a good jump. But at the normal discharge of 5.0 second feet for one conduit, with the second conduit closed, a large eddy was produced below the jump, which however was stopped by placing a 4-inch weir at the foot of the slope. This point was studied in more detail later.

Table 7.—Tail Water Velocities in Series A, Discharge 9.74 Second Feet

Run	Weir Height in Inches	Location of Weir	Surface Velocities in feet per second				Bottom Velocities in feet per second				Remarks
			South side	Center	North side	Mean	South side	Center	North side	Mean	
1604	....	No weir used	......	......	......	......	......	......	......	......	Unstable. No measurements.
1619	3¾	At foot of slope	......	......	......	......	......	......	......	......	" " "
1613	6	At foot of slope	1.66	1.66	1.77	1.60	1.20	0.45	1.12	0.70	Tail water even.
1606	7½	At foot of slope	2.05	1.44	1.84	1.56	1.32	0.37	1.42	0.74	" "
1622	9¼	At foot of slope	1.92	1.00	1.77	1.32	1.34	0.26	1.26	0.69	" "
1615	6 6	At foot of slope 36 inches below foot of slope	2.01	2.01	1.70	1.80	1.08	0.46	1.26	0.62	" "
1644	6	12 inches above foot of slope	1.77	1.77	1.92	1.76	0.82	0.41	0.96	0.60	" "
1642	6 6	12 inches above foot of slope 24 inches below foot of slope	1.92	1.84	1.70	1.57	0.96	0.38	0.85	0.48	" "
1639	3¾	24 inches above foot of slope	1.70	1.58	1.52	1.53	0.96	0.71	0.59	0.53	" "
1630	3¾	Same	1.70	1.64	1.70	1.57	0.76	0.57	0.82	0.59	" "
1624	3¾ 6	24 inches above foot of slope 12 inches below foot of slope	1.66	2.28	1.70	1.91	0.74	0.49	0.65	0.56	" "
1640	3¾ and 6	Same	1.64	2.52	1.38	1.92	0.76	0.39	0.67	0.47	" "

Table 8.—Tail Water Velocities in Series B, Discharge 9.74 Second Feet

Run	Wier Height in inches	Location of Weir	Surface Velocities in feet per second				Bottom Velocities in feet per second				Remarks
			South side	Center	North side	Mean*	South side	Center	North side	Mean*	
1651	....	No weir used	1.47 to 3.08	1.57	0.98	1.40	1.20	0.80	0.56	0.84	Unstable. Tendency to concentrate on south side.
1656	6	At foot of slope	1.58	1.12	1.52	1.34	1.17	0.39	1.26	0.80	Stable. Jump 1 foot above 1651.
1657	6	12 inches below foot of slope	1.64	1.12	1.70	1.39	1.34	0.28	1.10	0.75	Stable. Jump as in 1656.
1658	6	24 inches below foot of slope	1.77	1.03	1.92	1.44	1.17	0.30	1.38	0.79	Stable.
1659	3¾	8½ inches below foot of slope	1.64	1.22	1.58	1.42	1.30	0.31	1.17	0.77	Stable. Jump 0.5 foot above 1651.
1660	1¾	8½ inches below foot of slope	1.77	1.77	1.70	1.75	1.00	0.34	0.88	0.64	Stable. Jump as in 1651.
1662	{1¾ {3¾	8½ inches below foot of slope } 44½ inches below foot of slope }	1.84	1.77	1.52	1.72	0.98	0.29	0.96	0.63	Stable. Jump as in 1660.

* Mean velocities computed from those here recorded.
Jump straight in every case.

The above conclusions were checked by a few observations with a discharge of 12.0 second feet and in general by the observations of the later series.

Series B. The model was changed by shortening and steepening the slope, and by making other alterations as described above. The side rolls and center jet were eliminated, and the water approached the jump in a thin sheet of approximately uniform thickness. The jump began as before materially above the computed position, and ended below it, as shown in figure 58. In this and the following series, the jump was straight, was approximately at right angles to the center line of the channel, and was uniform in character. In fact it was the most perfect in appearance of all obtained in these experiments, except as explained below. With no obstructions in the channel, the tail water was satisfactory at discharges, equal in both conduits, of less than 9.74 second feet. But at this value there was a tendency for the flow to concentrate at one side, see figures 62 and 63.

Table 8 shows velocity measurements at a section 20 feet from the inlet end of the tank, with various weirs in use below the jump. The mean velocity at this section being about 1.10 feet per second, it is evident that the flow was not quite stable with no weirs at all, but that the velocities corresponded quite satisfactorily with the distribution to be expected in a natural flowing stream in the cases where a weir was employed. The bottom velocities were low in the center, but at the side they approximated the mean for the channel and were regarded as satisfactory. The best distribution seemed to be obtained with low weirs.

When one conduit was blocked, the long nose prevented the flowing water from spreading and the tail water flow was concentrated at one side of the channel at high velocity. The shortened slope also tended to prevent this discharge from spreading, and the channel was in this respect inferior to that of Series A. The use of a high weir was subsequently found necessary to baffle this concentration.

Series C. The effect of roughening the floor by the series of steps was to materially diminish the velocity above the jump. This was demonstrated by comparative velocity measurements with a Pitot tube, with and without the steps. As a consequence the flow below the jump was symmetrical and stable at all discharges if equal in both conduits, although the bottom velocity tended to be high in the center of the channel. This excess was

Table 9.—Tail Water Velocities in Series C, Stepped Slope, Discharge 9.74 Second Feet

Run	Weir Height in inches	Location of Weir	Surface Velocities in feet per second				Bottom Velocities in feet per second				Remarks
			South side	Center	North side	Mean	South side	Center	North side	Mean	
1663	....	No weir used	1.12	2.10	1.10	1.49	0.38	1.42	0.50	0.76	Jump straight in all runs.
1664	1¾	12 inches below foot of slope	1.22	1.84	1.34	1.58	0.74	0.37	0.88	0.56	....
1665	3¾	12 inches below foot of slope	1.70	1.05	1.84	1.36	1.05	0.33	1.00	0.68	....
1666	1¾	24 inches below foot of slope	1.03	2.10	1.12	1.76	0.63	0.82	0.67	0.69	....
1667	3¾	24 inches below foot of slope	1.70	1.05	1.64	1.31	1.16	0.34	0.88	0.77	....
1668	1¾	36 inches below foot of slope	1.10	2.10	1.08	1.65	0.43	0.96	0.57	0.60	....
1669	3¾	36 inches below foot of slope	1.77	1.17	1.92	1.39	1.30	0.33	1.03	0.78	....
1670	1¾	One 12 inches below, and one 36 inches below foot of slope	1.34	2.01	1.26	1.64	0.55	0.49	0.56	0.54	Distribution best.

Table 10.—Tail Water Velocities in Series D, Stepped Slope and "Stubby Nose," Discharge 9.74 Second Feet

Run	Weir Height in inches	Location of Weir	Surface Velocities in feet per second				Bottom Velocities in feet per second				Remarks
			South side	Center	North side	Mean	South side	Center	North side	Mean	
1671	1¾	One 12 inches below, and one 36 inches below foot of slope	0.90	2.52	1.70	1.89	0.82*	1.00	1.22	0.71	Eddy on south side at bottom.
1676	2½	12 inches below foot of slope	....	....	....	....	....	....	....	....	Not reduced, eddy on south side.
1672	2½	12 inches below foot of slope	1.17	2.21	1.92	1.84	0.57*	0.79	1.14	0.76	Eddy on south side at bottom. Regulating weir not level.
1677	3¾	12 inches below foot of slope	1.47	1.38	1.77	1.39	1.00	0.30	1.22	0.81	Symmetrical.
1674	3¾	12 inches below foot of slope and 6-inch baffle at foot of slope	1.34	2.01	1.84	1.78	0.58	0.32	1.30	0.61	Regulating weir not level.
1678	3¾	At foot of slope	1.77	2.10	1.84	1.76	0.92	0.27	1.08	0.59	....
1680	3¾	24 inches below foot of slope	1.58	1.08	1.92	1.39	1.14	0.29	1.58	0.87	....
1683	3¾	One 12 inches below, and one 36 inches below foot of slope	1.64	1.46	1.38	1.46	1.26	0.28	1.15	0.81	....
1679	5	24 inches below foot of slope	1.52	0.98	1.70	1.19	1.42	0.29	1.58	0.96	....
1681	3¾	12 inches below foot of slope	2.45	2.28	1.70	....	1.30	0.32	0.58	....	Tail water depth = 1.52. (Normal = 1.35.) Weir too low.

*Direction upstream.

Table 11.—Tail Water Velocities in Series E; Ogee Weirs located 12 inches below Foot of Slope. Velocities were measured at a section 17.0 feet from inlet end of tank

Run	Weir Height in inches	Discharge in second feet	Surface Velocities in feet per second				Bottom Velocities in feet per second				Remarks
			South side	Center	North side	Mean	South side	Center	North side	Mean	
1684	4½	9.74	1.42	2.32	1.84	2.02	0.34	0.56	0.79	0.57	....
1687	4½	9.74	1.77	2.32	1.26	1.88	0.85	0.71	0.55	0.67	....
1691	6	9.74	1.84	2.10	1.26	1.98	0.96	0.65	0.45	0.71	....
1695	8	9.74	2.21	1.58	1.92	1.83	1.20	0.38	0.65	0.59	....
1695a	8	9.74	2.21	1.70	1.84	1.86	1.34	0.39	0.51	0.60	....
1688	4½	9.74	2.45	2.32	1.70	2.08	1.12	0.55	0.92	0.75	8-inch certical weir 3 feet below 4½-inch weir.
1685	4½	7.1	*NR	1.25	3.15	....	*NR	Eddy	2.21	....	South conduit obstructed.
1692	6	6.5	*NR	1.37	3.53	....	*NR	*NR	2.32	....	" " "
1693	6	7.9	*NR	1.53	2.65	....	*NR	0.48	1.38	....	" " "
1696	8	6.7	0.92	1.16	1.77	1.28	0.44	0.30	0.71	0.45	" " "
1689	4½	6.8	1.05	2.10	2.21	1.94	0.42	0.56	1.05	0.62	" " " 8-inch vertical weir 3 feet below 4½-inch weir.
1686	4½	8.6	*NR	2.45	1.92	....	*NR	Whirl	1.05	....	South conduit obstructed.
1694	6	8.6	1.17	1.92	1.84	1.64	0.36	0.44	1.00	0.54	" " "
1697	8	8.6	1.70	1.52	2.32	1.61	0.67	0.36	1.00	0.55	" " "
1697a	8	8.6	1.64	1.47	2.10	1.56	0.69	0.34	1.00	0.54	" " "

* Direction upstream.
NR signifies that no velocity was read.

effectively eliminated by placing two low weirs below the foot of the slope, see table 9. However, the effect of blocking one conduit was the same as in Series B. An eddy was formed and high velocity was produced on one side of the channel, which was not satisfactorily prevented by a weir less than 4 inches high.

It was concluded that roughening the floor in this manner was desirable and that the method would be free from the impact stresses resulting from the use of small piers or boulders in the bottom.

Series D. The short nose used in this series results in a central jet as in Series A, see figure 50, requiring a higher weir to baffle the concentration than was needed in Series B and C. Table 10 indicates that nothing less than a 3¾-inch weir would be satisfactory, since eddies occur with lower weirs. This height of weir was fairly effective in checking eddies when one conduit was blocked, the water now having a better opportunity to spread. It was decided that a nose of medium length, intermediate betwen the other two, should be used, for the subsequent study.

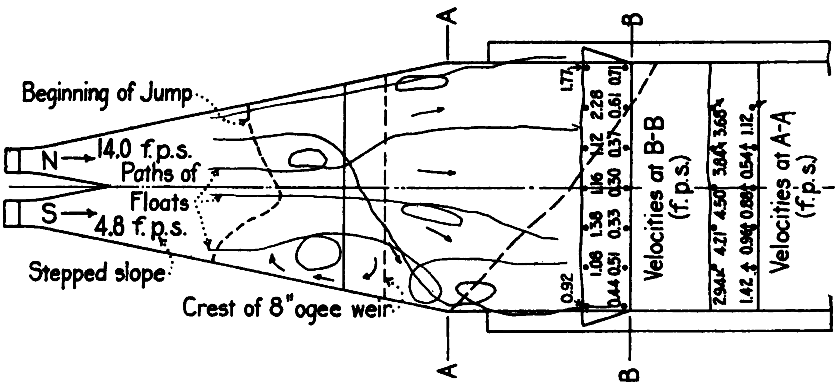


FIG. 64.—PLAN OF CHANNEL, SERIES E, SHOWING VELOCITIES BELOW THE JUMP, ONE CONDUIT PARTLY CLOSED.

Discharge in north conduit, 5.0 second feet; discharge in south conduit, 1.7 second feet; tail water depth, 1.18 feet; stepped slope, and an 8-inch ogee weir just below foot of slope.

Series E. It was by this time recognized that the height of the weirs, so far as the normal discharge is concerned, was not of first importance, and that minor variations in tail water velocities from this cause might be safely neglected. Inasmuch as weirs of ogee section would probably be used in construction, several heights of this shape were tried, especially to compare

their effects upon the discharge obtained when one conduit was partly closed.

Table 11 shows velocity measurements under various discharges and heights of weir, all the measurements being taken at the 17.0-foot section, 3.5 feet nearer the conduit outlet than heretofore. The 8-inch weir produced higher velocities at the sides than did the lower ones, indicating that the central excess is more thoroughly baffled; but the variations from the mean value of 1.10 feet per second did not seem to be excessive. With the south conduit obstructed so that it carried only about 40 per cent of the discharge in the north one, that is, with a combined discharge of 7.0 second feet, an eddy was produced below weirs less than 8 inches high with excessive discharge along the north

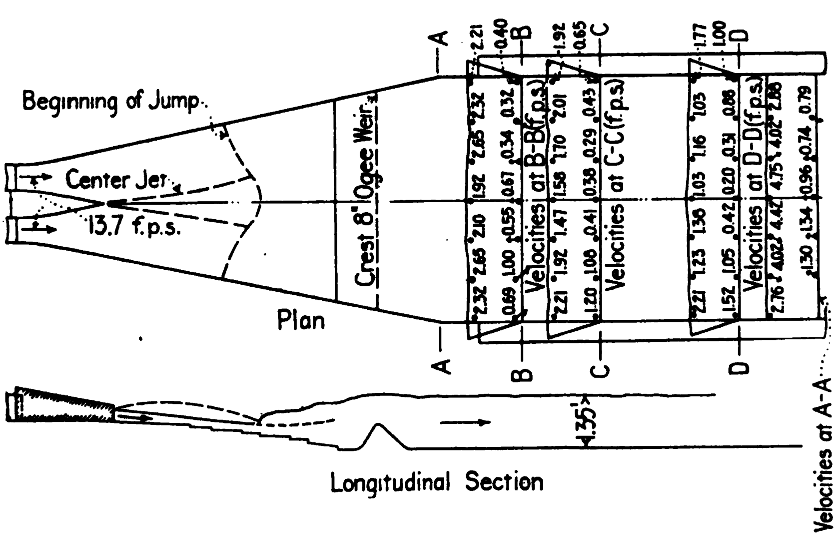


FIG. 65.—PLAN AND SECTION, SERIES E, SHOWING VELOCITIES BELOW THE JUMP.

Normal discharge of 9.74 second feet, 8-inch ogee weir just below foot of stepped slope.

side. With the obstruction lessened so as to permit the discharges of the two conduits to be in the ratio of about 75 to 100, that is with a combined discharge of 8.6 second feet, the 6-inch and 8-inch weirs gave satisfactory tail water conditions while the 4½-inch weir did not, see figures 64 and 65.

These observations indicated the advisability of using a weir not less than 8 inches high, or about half the depth of tail water needed to form the desired jump. But velocity measurements,

Table 12.—Tail Water Velocities in Series F; Floors at Various Levels below 8-inch Ogee Weir. (a) 8-inch ogee weir 12 inches below foot of slope. Velocities were measured at a section 17.0 feet from inlet end of tank

Run	Discharge in second feet	Height of Floor in inches	Height of 2nd Weir in inches	Surface Velocities in feet per second				Bottom Velocities in feet per second				REMARKS
				South side	Center	North side	Mean	South side	Center	North side	Mean	
1695	9.74	0	....	2.21	1.58	1.92	1.83	1.20	0.38	0.65	0.59	Mean velocity 1.09 feet per second.
1695a	9.74	0	....	2.21	1.70	1.84	1.86	1.34	0.39	0.51	0.60	" " " " " "
1698	9.74	3¾	....	2.01	2.10	1.38	1.92	0.96	0.54	0.49	0.56	" " 1.39 " " "
1698a	9.74	3¾	....	2.10	2.21	1.42	2.01	1.05	0.65	0.49	0.68	" " " " " "
1701	9.74	3¾	3¾	2.21	2.21	1.58	2.09	1.03	0.54	0.44	0.58	" " " " " "
1704	9.74	5¼	....	1.77	2.45	1.12	2.05	1.05	1.10	0.44	0.95	" " 1.61 " " "
1705	9.74	5¼	3¾	2.32	2.65	1.70	2.40	1.05	0.85	0.56	0.83	" " " " " "
1707	9.74	8	....	1.77	2.65	1.00	2.18	1.00	2.45	0.44	1.57	" " 2.11 " " "
1707a	9.74	8	2	2.10	2.45	1.47	2.12	1.05	1.38	0.57	1.10	....
1696	6.7	0	....	0.92	1.16	1.77	1.28	0.44	0.30	0.71	0.45	South conduit obstructed.
1699	7.0	3¾	....	*0.88	1.64	2.21	....	*.......	1.92	1.47	....	" " "
1703	7.0	3¾	3¾	0	1.84	2.65	1.82	*0.38	0.55	1.47	....	" " "
1706	7.0	5¼	3¾	0.82	2.01	2.32	1.63	0.69	1.10	1.58	0.97	" " "
1708	7.1	8	2	*0.92	2.76	2.94	....	*0.85	2.45	1.52	....	" " "
(b) 8-inch ogee weir 24 inches below foot of slope. Velocities were measured at a section 18.0 feet from inlet end of tank												
1709	9.74	5¼	....	1.77	2.01	1.23	1.77	1.70	1.58	0.69	1.32	Mean velocity 1.61 feet per second.
1710	9.74	5¼	3¾	1.77	2.10	1.42	1.88	1.38	0.96	0.92	0.97	" " " " " "
1713	9.74	5¼	3¾	2.21	2.10	1.42	2.01	1.56	0.76	0.85	0.98	" " " " " "
1715	9.74	5¼	3¾	2.10	2.45	1.42	2.11	1.52	0.85	0.85	0.91	Smooth slope.
1711	7.5	5¼	3¾	1.30	1.84	1.58	1.65	1.05	1.05	1.42	1.06	South conduit obstructed.
1712	5.0	5¼	3¾	0.69	1.47	1.84	1.29	0.58	1.10	1.42	1.02	" " closed.
1716	5.0	5¼	3¾	0.74	1.30	2.21	1.37	0.32	0.71	2.32	1.12	Same, smooth slope.

*Direction upstream.

with this height and the normal discharge, indicated considerable flow upstream in the bottom of the channel just below the weir. This suggested that the floor below the weir might be raised with economy. The idea was investigated in Series F.

Series F. In this set of experiments a satisfactory type of outlet was finally developed, and the discussion is therefore more detailed than heretofore. The results of the studies are condensed in table 12, where velocity measurements are given at a section 17.0 or 18.0 feet from the inlet end of the model tank. Unless otherwise designated all velocities are downstream and parallel to the channel axis. In several important cases the observations were checked by a repetition of the experiment. Reference should be made to the table for verification of the deduction below.

In all of the experiments there is a tendency toward greater velocities on the south side of the channel, believed to be due to a slight lack of symmetry in the central nose, which slightly unbalanced the sheet above the jump. This tendency may have been due, however, to irregularity in the series of steps on the slope, which became decidedly warped after use. As defects of this character might exist in the full-sized structure, the nose was not rectified. The existence of the irregularity should be borne in mind in studying the data.

When the floor below the 8-inch weir was raised 3¾, 5¼, or 8 inches, the distribution under the normal discharge of 9.74 second feet was symmetrical and showed no great variations from the mean velocities for the several depths, see experiments 1698, 1704, and 1707. Of course all velocities increased as the cross section of the channel diminished. The ability to raise the floor without losing the jump meant that much excavation and concrete could be saved in the full-sized structure.

Whenever, however, the tail water was lowered materially below the normal level, the jump was lost and the water was deflected into the air by the weir in the form of a sheet, see figures 66 and 67. A similar performance had been noted in Series A and elsewhere during the experiments, when the floor was not raised, but the tendency to so rise seemed to be greater here. This tendency was discouraged by moving the main weir downstream one foot, when the tail water could be lowered farther without losing the jump. A more effectual remedy was found, however, in a second weir, with crest elevations as high or higher than that of the first, and placed about three feet

below the latter. This tended to maintain the tail water level above the first weir, making the combination independent of variations in tail water levels below.

When the south conduit was partly blocked, an eddy tended to form below the single weir when the floor was raised. The second weir tended to check the eddy, and was most effective when the intermediate floor height of 5¼ inches was employed, see experiments 1706, 1711, and 1712, table 12. The effect is strikingly shown in figure 68, where the effluent water is shown going off without foam, although one conduit was completely closed. This type of construction, besides being economical and satisfactory in other respects, evidently solved the problem of unsymmetrical discharge.

In experiments 1715 and 1716 the steps were removed from the sloping floor, showing little change of velocities. But when the tail water was lowered to the minimum, by removing the regulating weir, and the discharge was increased to 12 second feet, the rising sheet of figure 67 was obtained even with the combination of weirs. The stepped slope is therefore an obstacle to such occurrence and may advantageously be retained. Figure

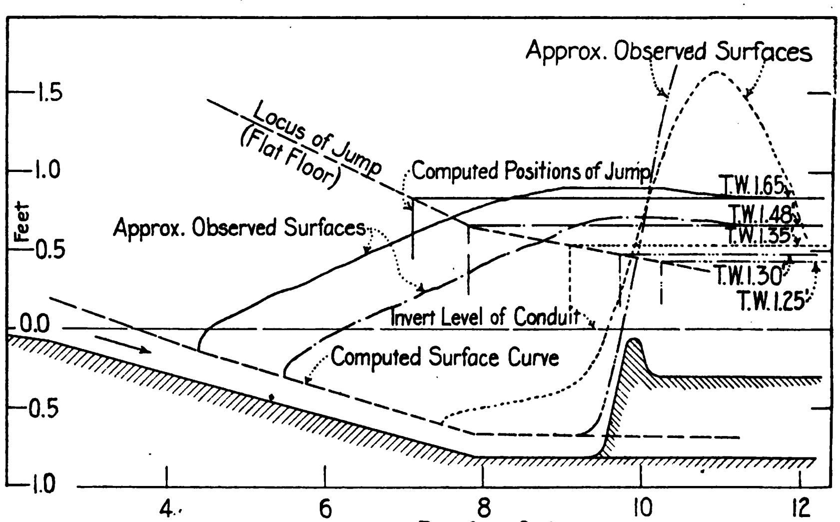


FIG. 66.—DIAGRAM SHOWING VARIOUS OBSERVED POSITIONS OF THE HYDRAULIC JUMP FOR DIFFERENT ELEVATIONS OF TAIL WATER, SERIES F.

Jump disappears and the water is sprayed into the air by the weir when the tail water becomes too low. Discharge 12 second feet.

FIG. 67.—VIEW SHOWING EFFECT OF TOO LOW TAIL WATER ELEVATION, SERIES F.

Discharge 12 second feet; both submerged weirs in place. The jump has disappeared and the water is sprayed high into the air in a continuous sheet at the upper weir. Smooth floor.

FIG. 68.—VIEW SHOWING SMOOTH, QUIET, UNIFORM CONDITION OF EFFLUENT BELOW SECOND WEIR, SERIES F.

One conduit carrying normal discharge of 5 second feet, other conduit completely blocked.

FIG. 69.—VIEW SHOWING SATISFACTORY OPERATION OF HYDRAULIC JUMP, SERIES F.

Uniform distribution of velocity in effluent; normal discharge of 9.74 second feet; 2 submerged weirs and raised floor.

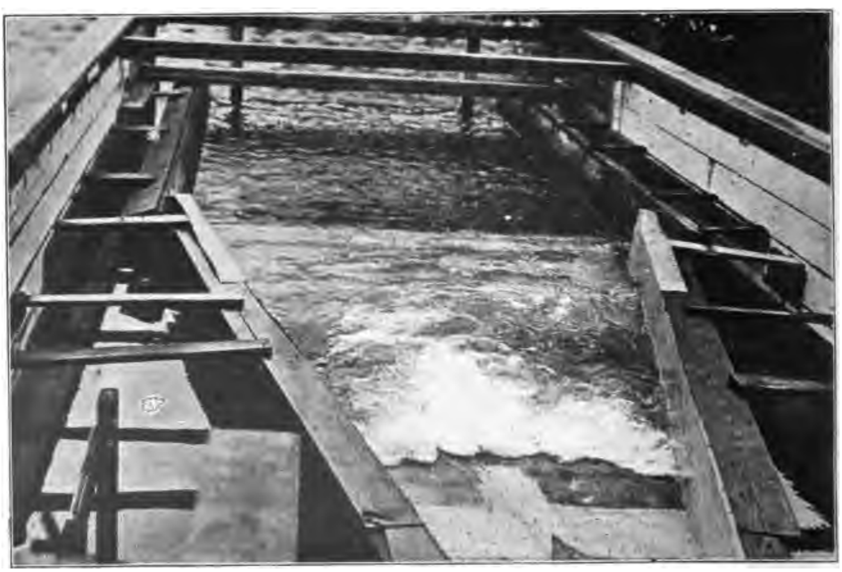


FIG. 70.—VIEW SHOWING OPERATION OF MODEL AS FINALLY ADOPTED, SERIES F.

Discharge of 3 second feet corresponding to an ordinary freshet. A small jump is formed.

66 shows in detail how the sheet was formed by lowering the tail water depth to 1.30 feet. This water was subsequently raised from below, to the depth of 1.25 feet without disturbing the sheet. It is evident that when the computed position, referred to the channel at the lower or original level, falls at the weir, the jump relation no longer obtains and therefore there will be no jump. It follows that it is the depth of water maintained upstream from the weir, rather than the weir itself, which produces the jump.

As a detail especially desirable in the outlets of the dams of the District, a central partition wall was inserted, so that it would be easy to shut off half of the channel during the low water season for inspection or repairs. It was found that with normal discharge and flow equal in the two conduits no particular effect occurred. But with one conduit blocked, it was necessary to make the wall high enough to keep the dead water on the idle side of the channel from spilling over into the jump. Otherwise an eddy was formed and the distributing effect of the weirs was diminished.

The experiments of Series F thus demonstrated a satisfactory type of outlet channel, whose characteristics may be summarized as follows:

1. It must have gradually expanding sides so shaped that there shall be no break in the contact between them and the water.

2. The floor should be steeply sloped where the jump is desired to occur, and should preferably be roughened by building it in steps. The limit of slope is the parabolic path of water issuing from the outlet as an orifice. On the other hand the drop should be decided so as to cause the sheet of water to cut under the water in the deep pool.

3. Far enough below the foot of the slope, to permit a jump to form under all circumstances of discharge, there should be a submerged weir whose crest is but little lower than the surface at low water and whose height is not less than one-half the tail water depth, just above the weir, at maximum discharge. Below this weir the channel may be more shallow. But a second baffling weir should be built below the first with its crest at least as high as that of the upper one. Below this point the depth must depend upon the capacity of the channel to resist erosion.

The behavior of the model embodying these conditions is shown in figures 69 and 70 and the form of the channel is shown

in figures 37 and 71. Reference should also be made to figure 72 which shows diagrams of velocity throughout the channel.

This form was the most satisfactory from all points of view, and the principles which it embodies have been applied in the design of the outlet channels of the several dams. It represents large economies over the tentative design, chiefly by virtue of the diminished length and the raised floor. Its features are generally applicable, and it is believed that with modifications it solves the problem of the elimination of energy under a wide range of conditions. It could, for instance, have been applied to the Croton Lake blow-off and to the Waldeck dam outlets, referred to in the introductory paragraphs, with assurance of satisfactory operation. Some of its features have been already adopted in the construction of ogee spillway dams, and the results of the experiments are therefore largely verified by existing experience.

SECTION XVIII.—USE OF BAFFLE PIERS

In the following pages is a discussion of some of the methods of eliminating energy which were tried and rejected. The entire series of experiments was begun with a form of channel marked

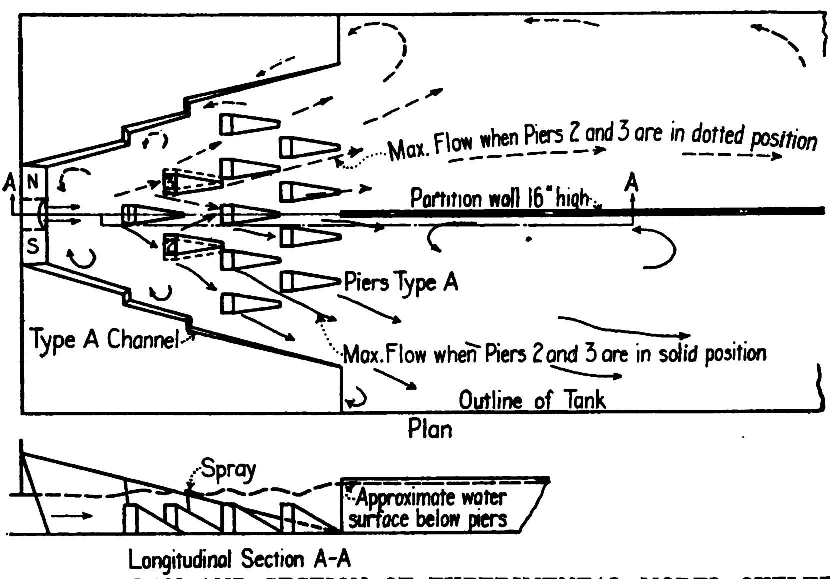


FIG. 73.—PLAN AND SECTION OF EXPERIMENTAL MODEL OUTLET CONTAINING BAFFLE PIERS.

Channel walls have offsets. Results show that the high velocity current shifts widely for a small change in the position of the piers.

FIG. 71.—VIEW OF MODEL AS FINALLY ADOPTED FOR USE.
Central longitudinal dividing wall installed.

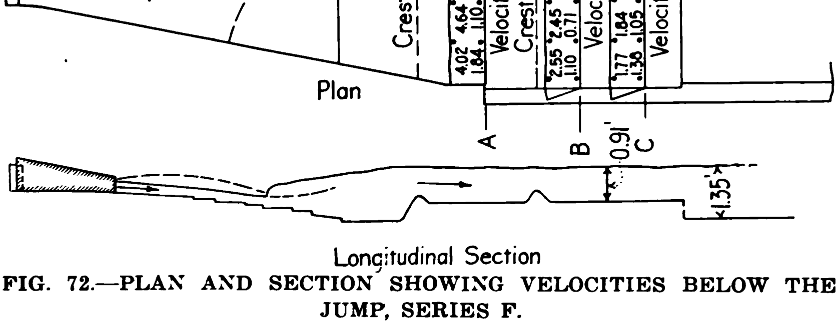


FIG. 72.—PLAN AND SECTION SHOWING VELOCITIES BELOW THE JUMP, SERIES F.

Stepped floor; 2 submerged weirs below jump; raised floor below upper weir; normal discharge of 9.74 second feet.

by offsets in the channel walls and a set of baffle piers, as shown in figure 73. By these devices it was hoped to obtain a well distributed flow with low velocity. This was found to be attainable, though with certain limitations, by modifying the form of channel and the arrangement of piers. Various conclusions from experiments upon the piers are summarized below:

1. The more nearly the side walls from the mouth of the outlet coincide with the shape that the free stream would take after leaving the outlet and before striking the piers, the better will the current be broken up by the piers. It was early found necessary to dispense with the offsets in the sides of the channel and to substitute the regular sides of figure 74. The chief defects of the method of baffling with piers were found to be instability of the effluent under slightly different conditions in the channel, and inability to predict with certainty what would occur under any given combination of piers, rate of discharge, and tail water level. This instability was materially reduced by the use of the second form of channel with regular sides.

2. The water is quieted in a shorter distance if caused to rise in the air, than if it is allowed to flow submerged into a body of

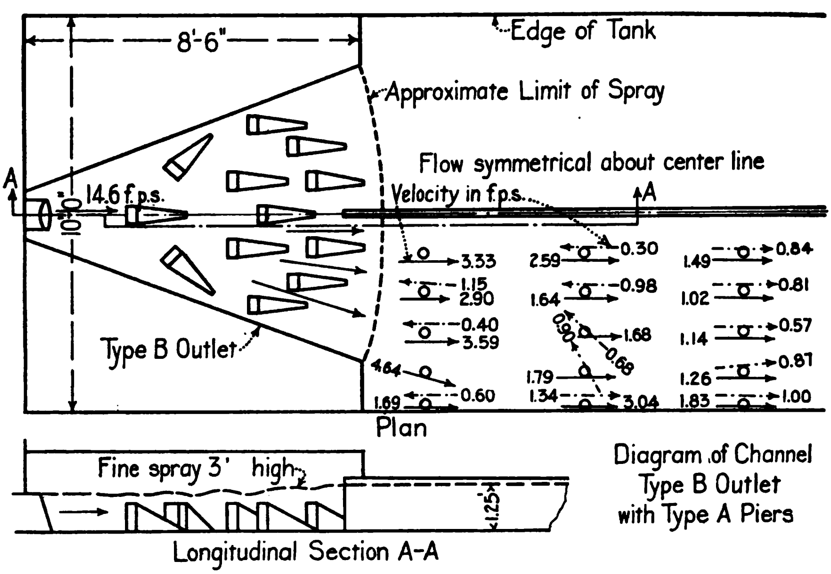


FIG. 74.—PLAN AND SECTION OF EXPERIMENTAL MODEL OUTLET CONTAINING BAFFLE PIERS, SHOWING VELOCITIES.

Expanding channel walls have no offsets. Full lines indicate surface velocities, broken lines indicate bottom velocities.

water; and the finer the spray caused in this way, the sooner the velocity and energy are dissipated. When the spray falls into the moving water which does not rise, the velocity of the latter is retarded.

3. The greater the depth of water below the piers, the less does the flow at high velocity spread, and the more completely is the spray prevented. This tends to concentration of the flow into small areas with only partial reduction of velocity. There is a pronounced tendency for this concentration to form a large whirl or eddy. The intensity and magnitude of this eddy are diminished materially by the use of a central wall such as shown in figure 74. Sometimes two smaller eddies are produced, but they are less objectionable than the single eddy.

4. Slight rearrangements of the piers lead to widely divergent effects upon the tail water, illustrated in figure 73, although this is perhaps an extreme case, and is no doubt affected by the offsets in the channel. Here a concentration of flow on the south side of the channel was transferred to the north side by shifting the piers numbered 2 and 3 outward from the center line one-half inch, the discharge being 8 second feet. The most satisfactory arrangement secured is that of figure 74, which shows a well distributed effluent under a tail water depth of 1.25 feet and discharge of 8 second feet. In this figure the solid arrows indicate surface velocity while the broken ones show bottom velocity. Pronounced vertical eddies are seen just below the piers.

5. It seems clearly possible, for any given conditions of discharge and tail water, to arrange a system of baffle piers which will reduce the velocity and produce a uniform effluent within a short distance below the piers. But the writers have been unable to discover any theoretic relationship which would govern the extension of such arrangement to other discharges and depths. Moreover, even in the model itself noticeable variations in performance were observed when the conditions for which any arrangement were found satisfactory were altered. Such devices therefore lack stability.

6. Severe impact stresses upon the piers would be produced, necessitating heavy reinforcement in the bottom of the channel and heavy anchorage weights for the piers. The magnitude of these stresses is difficult to determine even approximately, especially those which arise from the transverse pressures of the eddy currents. Were the device perfectly satisfactory in respect

FIG. 75.—VIEW AT RIGHT ANGLES TO THE CURRENT SHOWING THE WATER SPRAYED INTO THE AIR BY A ¾-INCH BOARD.

Painted lines divide floor of channel into one-foot squares; moving stream before striking board was but little thicker than board; no back-water from regulating weir below.

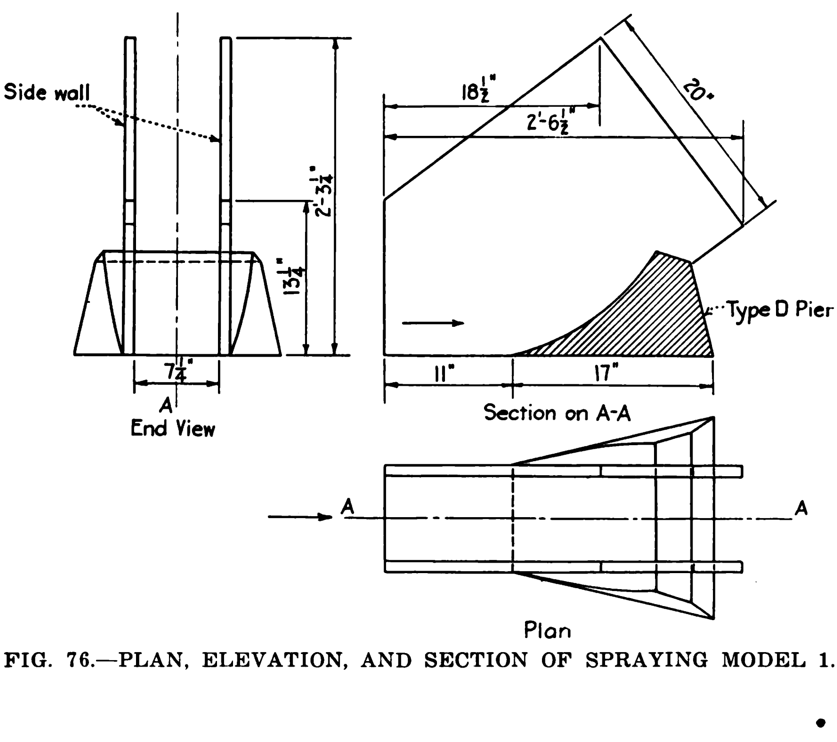


FIG. 76.—PLAN, ELEVATION, AND SECTION OF SPRAYING MODEL 1.

FIG. 78.—VIEW SHOWING 12 SECOND FEET BEING SPRAYED INTO AIR BY SPRAYING MODEL 2.

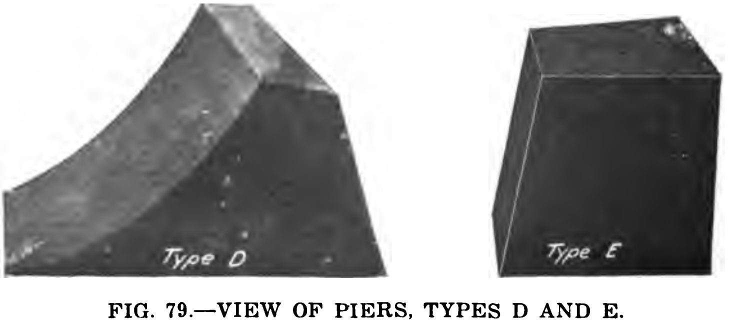


FIG. 79.—VIEW OF PIERS, TYPES D AND E.

to its hydraulic behavior, heavy expenditures would be justified in order to make the structures safe, but not otherwise.

7. It was therefore concluded that the use of baffle piers in the channel floor did not offer a satisfactory solution to the problem. They might, however, be applicable in a case where the discharge and depth were not subject to variation, although, even then it would be necessary to determine the proper arrangement of piers for the peculiar conditions of the channel.

SECTION XIX.—USE OF SPRAYING DEVICES

The observations upon piers suggested that devices might be employed which would operate primarily by deflecting the water into the air so as to cause it to form spray and fall into a pool below. The ability to do this was recognized, but the effect of the falling water in the pool below was uncertain.

It was easy to cause the water to rise from the horizontal floor. In figure 75 is shown the effect of placing a ¾-inch board on the floor 4 feet below the outlet. The water had spread into a thin sheet, not much thicker than the board, before reaching the latter, and there was no backwater from the outlet weir. Were

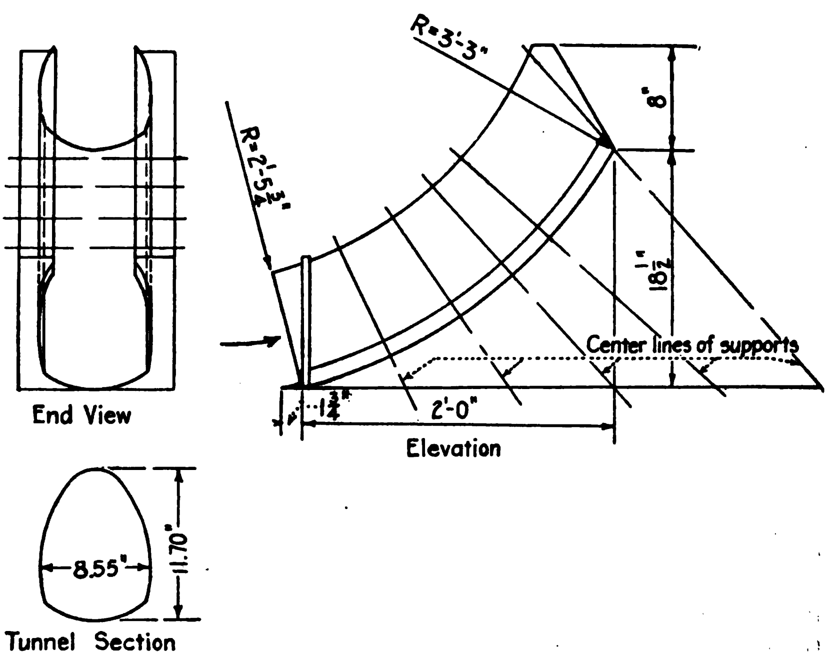


FIG. 77.—ELEVATIONS OF SPRAYING MODEL 2.

either of these conditions otherwise, the performance of the water would be quite different, as will be shown later. The same effect was obtained with weirs of greater height.

In order to study the performance of thick sheets of water, the two spraying models of figures 76 and 77 were tried. Both of these were placed on the floor of the tank as close to the conduit outlet as practicable—in fact with the sides overlapping the sides of the latter, so as to prevent the water from escaping in any other way than over the model. The effect of these is shown strikingly in figure 78, where the maximum discharge of the plant, 12 second feet, is being sprayed into the air.

In the operation of model 1, a depth of 1 foot was maintained in the model tank below the outlet, while the discharge was varied from 4 to 10 second feet. For the smaller discharge the flow just spilled over the lip of the model and then flowed downstream in the center of the channel. With the discharge at 10 second feet, and conduit velocity 18.2 feet, the water sprayed 4.3 feet high above the floor, and fell about 14 feet from the conduit over a width of 3 feet. At discharges of 7 second feet or more, when the height of spray was about 3.1 feet, the impact of the falling water was observed by placing the hand at the bottom of the pool. When the depth of tail water was raised to 1.4 feet, this impact could no longer be felt, indicating that the energy of the falling water was absorbed before reaching the bottom.

The study of this model shows that it is possible to throw water into the air in a solid sheet, and that whenever the tail water surface is high enough to come into contact with the rising stream the water will be broken up into spray so that it does not strike the tail water as a solid stream. All of these observations were checked by others upon model 2.

The next study was made of a single pier designated as type D, figure 79, placed in the channel with spreading walls. When placed close to the outlet, but free from contact with the channel walls, it caused the discharge to spread as well as to rise from its concave surface, the water thus being spread in a fan-shaped manner. Although considerable water escaped at high velocity past the pier, it was considered objectionable to extend the pier to the channel walls. In the full-sized structure, such an arrangement would submerge the conduits at low discharges, rendering it difficult to inspect or repair them. The idea of dropping the floor had not been developed at this time.

FIG. 80.—VIEW SHOWING SPRAY AND CURRENTS PRODUCED BY USING A SINGLE CENTRAL PIER OF TYPE D AND TWO ROWS OF LOWER PIERS OF TYPE E.

Discharge 8 second feet; no regulating weir.

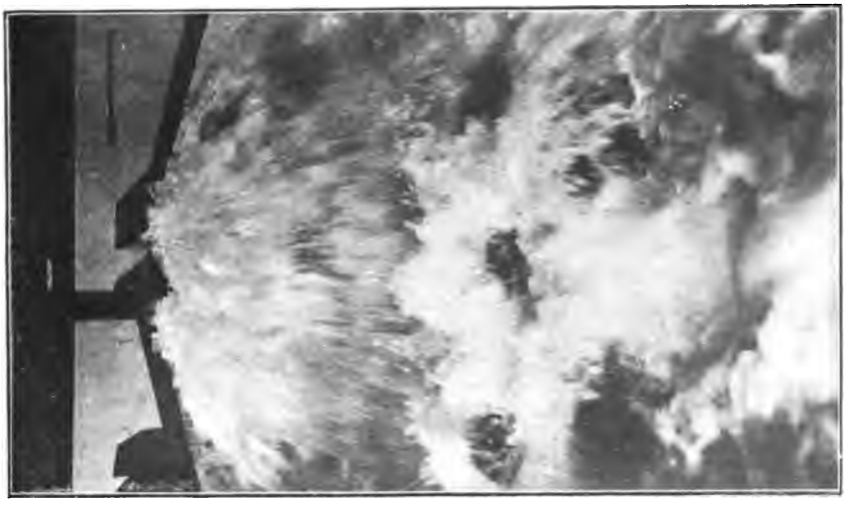


FIG. 81.—VIEW SHOWING SPRAY AND CURRENTS PRODUCED BY COMBINATION OF TYPE D AND TYPE E PIERS.

Discharge 8 second feet; tail water depth 1 foot.

The effluent from pier D was reasonably even, although large velocities were noted at the sides of the channel. To obtain greater uniformity, another combination was tried, consisting of a single pier of type D and a row of smaller piers of type E, figure 79, so placed as to baffle the fan-shaped sheet from the upper weir. This arrangement was found to be effective for a wide range of tail water depths, but to be less useful under small discharges, because in this case the small piers were too far away from the foot of the jet. Figures 80 and 81 show the disturbance created by the combination of piers. With a discharge of 8 second feet, and no backwater from the regulating weir, the velocities below the piers were higher at the sides than in the center, due partly to the escape of a portion of the discharge around the sides of the central pier D. As the tail water was increased by raising the regulating wear, the distribution improved, although there was always some excess at the sides.

Pier D proved to possess great stability, the vertical component of the forces exerted upon it exceeding the horizontal component of the latter. This was true even under conditions of maximum discharge, and no fastening to the floor of the experimental tank was, therefore, required to maintain pier D in position. There was, however, the same difficulty in securing a satisfactory anchorage for the lower piers that applied to the set of piers first described, referred to in paragraph 6 of Section XVIII.

The study of this device was carried no further. At this stage in the experiments the method of utilizing the hydraulic jump had become most promising, and that line of investigation was taken up.

In connection with the general subject, however, some studies were made of the paths of water as deflected from small weirs in the bottom of the channel, inclined at various angles to the bottom. These are chiefly of value as indicating the shape of the sheets which may be expected with various proportions of depth, velocity, and weir height.

Weirs were used whose inclination to the horizontal was 45, 60, and 90 degrees. They extended over a material part of the width of the channel and the section on the center line of the channel was taken as characteristic. The depth of water above the weirs had a most pronounced influence upon the behavior of the water as is shown in the diagrams, figures 82 and 83.

The observations showed that when the sheet was thin and the weir relatively high, the top of the sheet was above the

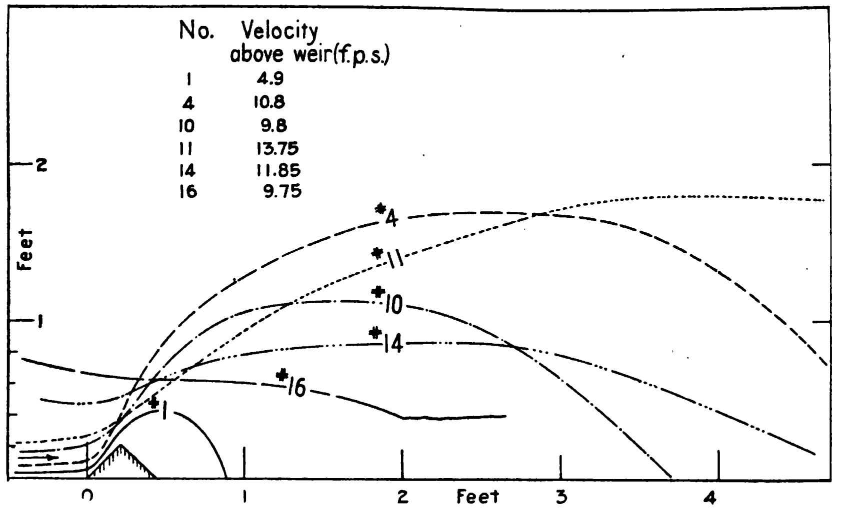


FIG. 82.—DIAGRAM SHOWING PROFILES OF SPRAY.

Currents of various depths and velocities striking a weir with upper face inclined 45° to the horizontal.

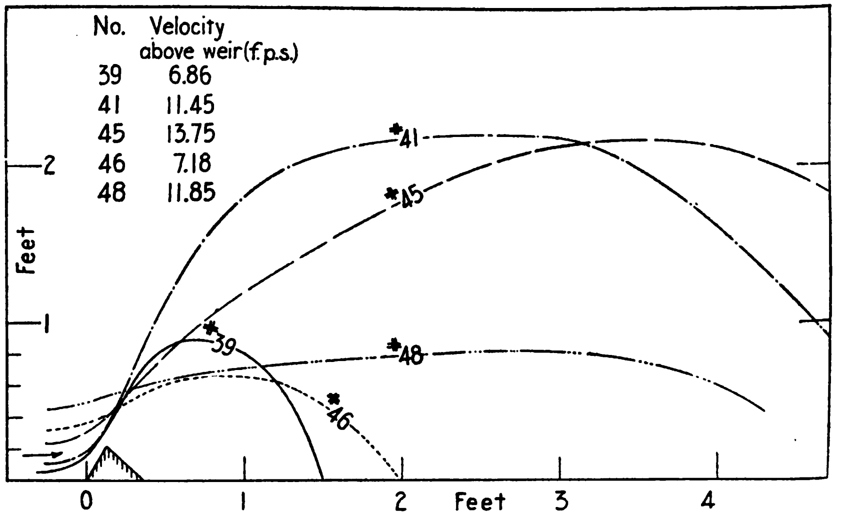


FIG. 83.—DIAGRAM SHOWING PROFILES OF SPRAY.

Currents of various depths and velocities striking a weir with upper face inclined 60° to the horizontal.

tangent to the weir. As the thickness of the sheet increased, other conditions remaining equal, the top of the sheet became tangent to a line parallel to the surface of the weir. As the depth above the weir became greater the sheet diminished in height and range, and finally with great enough depth the weir produced no visible effect upon the surface. Figures 82 and 83 illustrate these remarks. An attempt was made to formulate laws for the height and range of these sheets but the number of observations

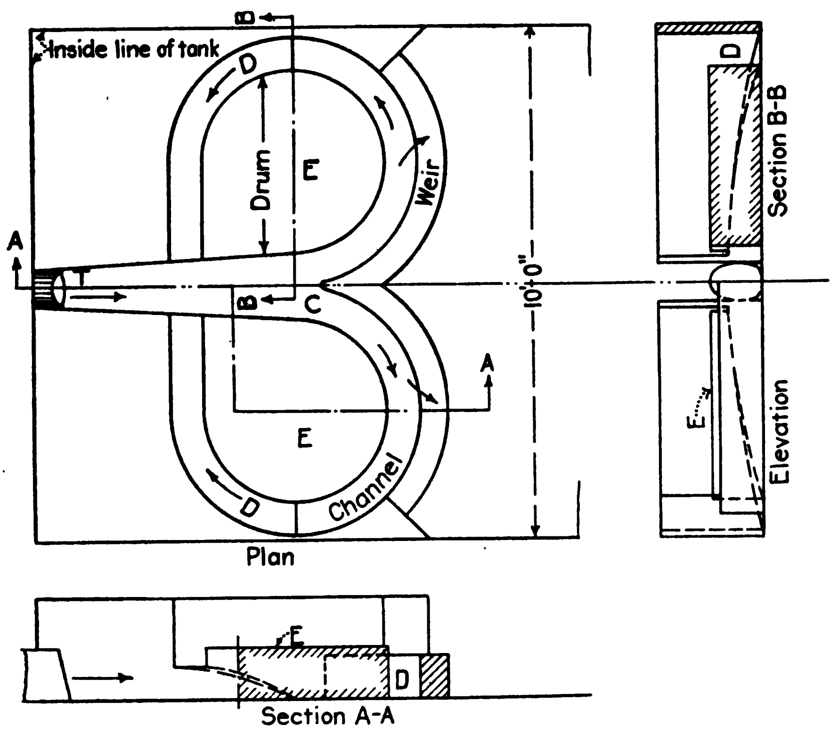


FIG. 84.—PLAN AND SECTIONS OF REVERSED CURRENT MODEL.

were too few and the variables involved were too numerous to allow a satisfactory conclusion.

The behavior of these sheets was also much affected by the depth of the tail water, although this is not shown in the diagrams. In the cases shown the tail water was kept low to avoid interference. But when it was increased by raising the regulating weir, the rise of the sheet was diminished. By increasing the tail water sufficiently the rise was submerged or drowned, and a hydraulic jump was formed above the weir in a manner some-

what similar to that finally worked out in the experiments on the jump. In fact, this suggested some of the steps tried in the later experiments.

SECTION XX.—REVERSED CURRENT MODEL

This was a device intended to combine the effects of spraying into the atmosphere, direct impact, and other kinds of friction in the elimination of energy. Referring to figure 84, the water leaving the outlet *T*, gradually transforms its cross section to a trapezoidal one in the channel until it reaches the point of the

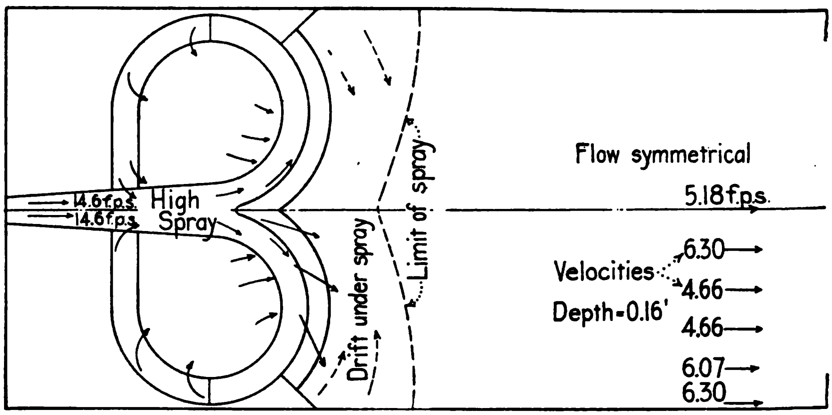


FIG. 85.—DIAGRAM SHOWING EFFLUENT VELOCITIES FROM REVERSED CURRENT MODEL.

Discharge 8.2 second feet; tail water depth of 0.16 feet.

curved weir, *C*. This point divides the stream into two parts which follow around the inner face of the curved weirs, part of the water rising and flowing over the weirs into the channel below. The rest flows up the inclined floor of the passage *D*, the two streams meeting over the stream from the outlet and falling upon it. This water is then carried down again by the main stream with the effect of reducing the velocity of the latter. In some experiments the drums *E*, were omitted.

The principal investigation of this model was made with a constant discharge of 8.2 second feet and a velocity at the conduit mouth of 14.6 feet per second, the depth of water below the model being varied by the use of regulating weirs of different heights. Some additional observations were made at other discharges.

FIG. 86.—VIEW SHOWING OPERATION OF REVERSED CURRENT MODEL.

Tail water depth 0.10 feet.

FIG. 87.—VIEW SHOWING OPERATION OF REVERSED CURRENT MODEL.

Tail water depth 1.80 feet.

The general result of the use of this form of model is an excess of velocity at the sides of the channel. When the depth below is small so that the spray over the curved weir forms and falls freely, this difference is small, but the velocities across the channel are high, see figures 85 and 86. As the depth below the model

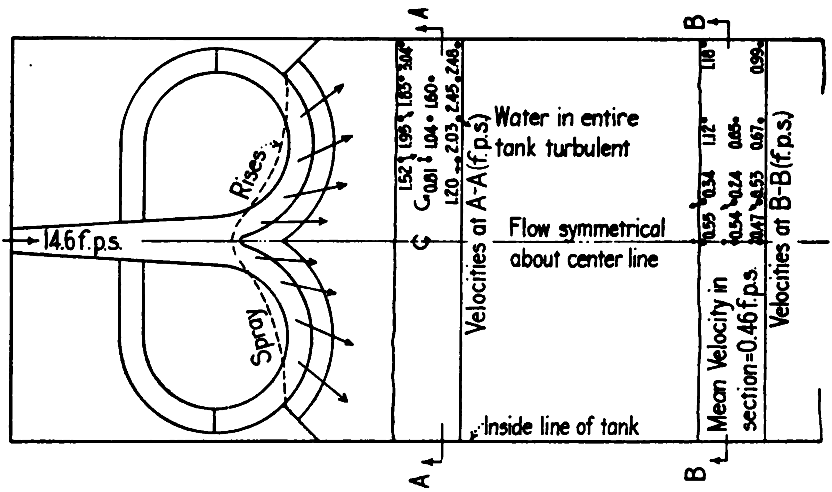


FIG. 88.—DIAGRAM SHOWING EFFLUENT VELOCITIES.

Reversed current model; discharge 8.2 second feet; tail water depth 1.79 feet.

increases, the difference between center and side velocities below the model also increases. When the surface below rises materially above the crest of the circular weirs, the center velocities are upstream and the discharge travels down the sides of the channel. Inasmuch as the water below the model must be deep in order to secure a low mean velocity of effluent, this condition is unsatisfactory, see figures 87 and 88.

This condition could be avoided by raising the weirs so as to keep them well above tail water level, but in the design of the conduits it was held to be inadvisable to have the weirs much higher than the conduit invert. These considerations together with the excessive cost of this method upon the large scale, caused it to be rejected. Curiously, in the form of channel finally worked out for the control of the hydraulic jump the surface flow of the water somewhat resembled that of this model, without the use of the complicated and expensive channels. It might be remarked that the performance of the model without the drums *E*, of figure 84, was similar to that with the drums, except that

there was greater irregularity in the disturbance and in the effluent.

This method of control and those described in the two preceding Sections were not studied in close detail, because they were deemed to be decidedly inferior to that of the jump.